AP® CALCULUS AB
2015 SCORING GUIDELINES

Question 6

Consider the curve given by the equation $y^3 - xy = 2$. It can be shown that $\frac{dy}{dx} = \frac{y}{3y^2 - x}$.

(a) Write an equation for the line tangent to the curve at the point $(-1, 1)$.

(b) Find the coordinates of all points on the curve at which the line tangent to the curve at that point is vertical.

(c) Evaluate $\frac{d^2y}{dx^2}$ at the point on the curve where $x = -1$ and $y = 1$.

2005 #5 Form B

Consider the curve given by $y^2 = 2 + xy$

a. Show that $\frac{dy}{dx} = \frac{y}{2y - x}$.

b. Find all points (x, y) on the curve where the line tangent to the curve has slope ½.

c. Show that there are no points (x, y) on the curve where the line tangent to the curve is horizontal.

2000 #5

Consider the curve given by $xy^2 - x^3y = 6$.

a. Show that $\frac{dy}{dx} = \frac{3x^2y - y^2}{2xy - x^3}$.

b. Find all points on the curve whose x-coordinate is 1, and write an equation for the tangent line at each of these points.

c. Find the x-coordinate of each point on the curve where the tangent line is vertical.

2001 #6

The function f is differentiable for all real numbers. The point (3, ¼) is on the graph of y = f(x), and the slope at each point (x, y) on the graph is given by $\frac{dy}{dx} = y^2(6-2x)$.

a. Find $\frac{d^2y}{dx^2}$ and evaluate it at the point (3, ¼).

The slope of the tangent is –1 at the point (0,1) on $x^3 - 6xy - ky^3 = a$, where k and a are constants. The values of the constants a and k are:

A. k = 1, a = –1 B. k = 2, a = –2 C. k = 3, a = –3 D. k = –2, a = 4. E. k = –1, a = 3

AP Free Response
Implicit Differentiation

Name _____

1998 #6

Consider the curve defined by $2y^3 + 6x^2y - 12x^2 + 6y = 1$

a. Show that $\frac{dy}{dx} = \frac{4x - 2xy}{x^2 + y^2 + 1}$.

b. Write an equation of each horizontal tangent line to the curve.

c. The line through the origin with slope –1 is tangent to the curve at point P . Find the x- and y-coordinates of point P.

1992 AB 4:

Consider the curve defined by the equation $y + \cos y = x + 1 \quad for \quad 0 \le x \le 2\pi$

(a) Find $\frac{dy}{dx}$ in terms of y.

(b) Write an equation for each vertical tangent to the curve.

(c) Find $\frac{d^2y}{dx^2}$ in terms of y.

WS On Implicit Differentiation Name_____

Find the implicit derivative:

1. $x^2 - xy - y^3 = xy^2$

2. $\sqrt{x} + \sqrt{y} = 25$

3. $\cos^2 y + \sin^2 y = y + 2$

4. $\sin(xy) = 2x + 5$

Write the equation of the tangent line to the following curves at the indicated point.

5. $xy^2 = 1$ at (1,-1)

6. $\frac{x^2}{xy-4} = y^2$ at (4,2)

Find the second derivative y'' or $\frac{d^2y}{dx^2}$

7. $y^2 + 2y = 2x + 1$

8. $2xy = y^2$

9. A particle moves according to a law of motion $s = t^3 - 6t^2 + 9t + 11$.

   a. What is the velocity of the particle at t=0.

   b. During what intervals is the particle moving left?

   c. What is the total distance travelled by the particle from t=o to t=2.

   d. Determine the displacement of the particle from t=0 to t=2.

10. A dynamite blast blows a heavy rock straight up with a launch velocity of 160 ft/sec. It reaches a height of $s = 160t - 16t^2$ How high does the rock go? How fast is the rock going when it is 256 ft above the ground?