Notes on Related Rates

In a related rates problem the idea is to compute the rate of change of one quantity in terms of the rate of change of another.

The procedure is to find an equation that relates the two quantities and then use the chain rule to differentiate both sides with respect to time.

STRATEGY

1. Read problem carefully
2. Draw a diagram if possible
3. Introduce notation. Assign symbols to all quantities that are functions of time.
4. Express given info and the required rate in terms of derivatives (ex: dv/dt)
5. Write an equation that relates the various quantities of the problem. If necessary, use geometry to eliminate one of the variables by substitution
6. Use chain rule to differentiate both sides with respect to time
7. Substitute given info in resulting equation and solve for the unknown

NOTE: Do not put in any given quantities unless it is fixed-not changing. (ex: ladder is ten feet tall; this does not change)

NOTES ON RELATED RATES

Example #1

Air is being pumped into a spherical balloon so that its volume increases at a rate of $100cm^3/s$. How fast is the radius of the balloon increasing when the diameter is 50 cm?

Example #2

A 10 foot ladder rests against a vertical wall. If the bottom of the ladder slides away from the wall at a rate of 1 ft/s, how fast is the top of the ladder sliding down the wall when the bottom is 6 feet from the wall?

Example #3

A water tank has the shape of an inverted circular cone with base radius 2m and height 4m. If water is being pumped into the tank at a rate of $2m^3/min$ find rate at which the water level is rising when the water is 3m deep.

Example #4

Car A is traveling west at 50 mi/h and Car B is traveling north at 60 mi/h. Both are headed for the intersection of the roads. At what rate are the cars approaching each other when A is 0.3 miles and B is 0.4 miles from the intersection?

Example #5

A man walks along a straight path at a speed of 4 ft/s. A search light is located on the ground 20 feet from the path and is focused on the man. At what rate is the searchlight rotating when the man is 15 ft down the path?