Optimization Problems

Steps in solving problems:

1. Understand the problem. What is the unknown? What's given?
2. Draw a diagram.
3. Introduce notation. Assign symbols to the quantity to be max or min. Label all other givens.
4. Express unknown in terms of givens.
5. If unknown is expressed as a function of more than one variable, use given info to find relationship among the variables. Then use these equations to eliminate all but one of the variables.
6. Find absolute max or min value. In particular, if the domain of f is closed used the closed interval method.

Example One:

A farmer has 2400 feet of fencing and wants to fence off a rectangular field that borders a straight river. He needs no fence along the river. What are the dimensions of the field that has the largest area?

Example Two:

A cylindrical can is to be made to hold 1 liter of oil. Find the dimensions to minimize the cost of the metal to manufacture the can. (Minimize surface area)

Example Three:
Which points on the graph of $y = 4-x^2$ are closest to $(0,2)$?

Example Four:
A man launches his boat from point A on a bank of a straight river, 3 km wide, and wants to reach point B 8 km downstream on opposite bank as quickly as possible. (Assume speed of water is neglibile compared to speed of rowing) He rows at 6 km/hr and he runs at 8 km/hr.

Example Five:
Find the area of the largest rectangle that can be inscribed in a semicircle of radius r. (There's an alternate solution to this involving trig.)