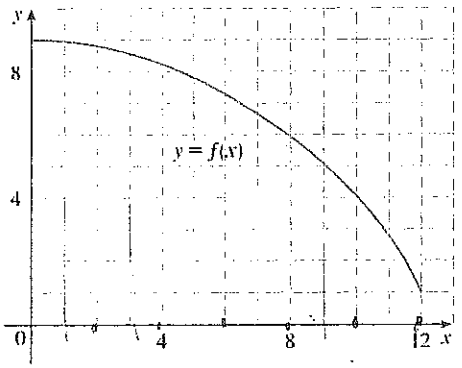
Review for Test on Riemann Sums / Basic Integration

Name _____

(a) Use six rectangles to find estimates of each type for the area under the given graph of f from $x = 0$ to $x = 12$.



(i) L_6 (sample points are left endpoints)
(ii) R_6 (sample points are right endpoints)
(iii) M_6 (sample points are midpoints)

(b) Is L_6 an underestimate or overestimate of the true ar
(c) Is R_6 an underestimate or overestimate of the true ar

(d) T_6 =

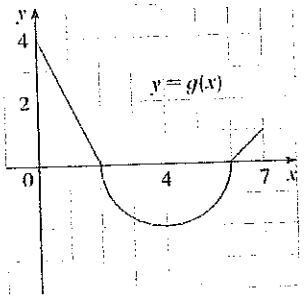
1. Evaluate the Riemann sum for $f(x) = 2 - x^2$, $0 \le x \le 2$, with four subintervals, taking the sample points to be right endpoints. Explain, with the aid of a diagram, what the Riemann sum represents.

7. A table of values of an increasing function f is shown. Use the table to find lower and upper estimates for $\int_0^{25} f(x)\,dx$.

x	0	5	10	15	20	25
$f(x)$	−42	−37	−25	−6	15	36

30. The graph of g consists of two straight lines and a semicircle. Use it to evaluate each integral.

(a) $\int_0^2 g(x)\,dx$ (b) $\int_2^6 g(x)\,dx$ (c) $\int_0^7 g(x)\,dx$



43–44 □ Write the given sum or difference as a single integral in the form $\int_a^b f(x)\,dx$.

43. $\int_1^3 f(x)\,dx + \int_3^6 f(x)\,dx + \int_6^{12} f(x)\,dx$

44. $\int_2^{10} f(x)\,dx - \int_2^7 f(x)\,dx$

45. If $\int_2^8 f(x)\,dx = 1.7$ and $\int_5^8 f(x)\,dx = 2.5$, find $\int_2^5 f(x)\,dx$.

46. If $\int_0^1 f(t)\,dt = 2$, $\int_0^4 f(t)\,dt = -6$, and $\int_3^4 f(t)\,dt = 1$, find $\int_1^3 f(t)\,dt$.

5–16 □ Use Part 1 of the Fundamental Theorem of Calculus to the derivative of the function.

9. $F(x) = \int_x^2 \cos(t^2)\,dt$

12. $h(x) = \int_0^{x^2} \sqrt{1 + r^3}\,dr$

17–34 □ Use Part 2 of the Fundamental Theorem of Calculus to evaluate the integral, or explain why it does not exist.

25. $\int_3^3 \sqrt{x^5 + 2}\,dx$

$\int_{\pi/4}^{\pi/3} \sin t\,dt$

32. $\int_{\pi/4}^{\pi} \sec^2\theta\,d\theta$

8. $\int x(1 + 2x^4)\,dx$

24. $\int_1^2 \frac{x^2 + 1}{\sqrt{x}}\,dx$

26. $\int_1^2 \left(x + \frac{1}{x}\right)^2 dx$

34. $\int_{-5}^{-2} \frac{x^4 - 1}{x^2 + 1}\,dx$

1–8 □ Find the average value of the function on the given interval.

1. $f(x) = x^2$, $[-1, 1]$

3. $g(x) = \cos x$, $[0, \pi/2]$

56. $\int_{-e^2}^{-e} \frac{3}{x}\,dx$

59. $\int_1^{\sqrt{3}} \frac{6}{1 + x^2}\,dx$

66. $\int \frac{1}{x\sqrt{x^2 - 4}}\,dx$