AP Calculus
3.3 Worksheet Day 1

All work must be shown in this course for full credit. Unsupported answers may receive NO credit.

1. A derivative tells you the _____ of a function.

2. What is the power rule for derivatives? (i.e. how do you take the derivative of $y = x^n$?)

3. For each of the following functions, find $\frac{dy}{dx}$.

a) $y = -2x^3 + x$

b) $y = \frac{x^4}{3} - \frac{x^2}{7} + 5$

c) $y = \frac{5}{x^2} + \frac{6}{x} - 8x^3$

d) $y = \frac{x^{-3}}{2} + 5x^{-4} - 3x^{-6}$

e) $y = 5x^4 + 2x^3 - 8x^2 - 7x + 11$

f) $y = 7x - 8$

g) $y = (x^2 - 3)(x + 4)$

h) $y = \frac{x^5 - 2x^4 + 3x^3}{x^5}$

i) $y = \sqrt{x} + \frac{3}{\sqrt{x}} - 6x^{5/3} + \frac{7}{x^3}$

4. [Calculator Required] We want to find all points where the graph of $y = x^4 - 5x^3 - 3x^2 + 13x + 10$ has a horizontal tangent line.

a) First, find an equation for y'.

b) A horizontal tangent line will have a slope = _____. So set $y' =$ _____, and use your calculator to solve this equation.

5. Find the equation of the tangent line to the function $y = \dfrac{x^2 + x - 2}{2x}$ at the point where $x = 1$.

6. Find the equation of the normal line to the function $y = x^3 - 5\ x + 1$ at the point when $x = 2$.

7. Find the points on the curve $y = x^3 + 3x^2 - 9x + 7$ where the tangent line is parallel to the x-axis.

8. Consider the curve $y = x^3 + x$.

   a) Find the tangents to the curve at all the points where the slope is 4. (be careful! … it doesn't say $x = 4$!)

   b) What is the smallest slope of the curve? At what value of x does the curve have this value?

9. Find the x- and y-intercepts of the line that is tangent to the curve $y = x^3$ at the point $(-2, -8)$.

10. If the line normal to the graph of f at the point $(1, 2)$ passes through the point $(-1, 1)$, then which of the following gives the value of $f'(1)$?

A	–2
B	2
C	–1/2
D	1/2
E	3

AP Calculus
3.3 Worksheet Day 2

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1. What is the product rule?

2. What is the quotient rule?

3. Let $f(x)=\left(3x^3+4x^2\right)\left(2x^4-5x\right)$.

   a) Find $f'(x)$ without using the product rule

   b) Find $f'(x)$ using the product rule.

4. Let $f(x)=\dfrac{x^2+4}{x}$.

   a) Find $f'(x)$ without using the quotient rule

   b) Find $f'(x)$ using the quotient rule.

5. Find $\dfrac{dy}{dx}$ for each of the following functions.

   a) $y=\dfrac{2x-5}{3x+2}$

   b) $y=(3-x)\left(2+x^2\right)^{-1}$

   c) $y=\dfrac{x^3}{8-x^2}$

6. For $a-d$, write a expression for $f'(x)$ and then use it to find $f'(2)$ given the following information:

$$g(2)=3 \quad g'(2)=-2$$
$$h(2)=-1 \quad h'(2)=4$$

a) $f(x)=2g(x)+h(x)$

b) $f(x)=4-h(x)$

c) $f(x)=g(x)h(x)$

d) $f(x)=\dfrac{g(x)}{h(x)}$

7. Suppose u and v are differentiable functions of $x=3$ and that $u(3)=4$, $\left.\dfrac{du}{dx}\right|_{x=3}=-3$, $v(3)=2$, and $\left.\dfrac{dv}{dx}\right|_{x=3}=3$. Find the values of the following derivatives at $x=3$.

a) $\dfrac{d}{dx}\left(\dfrac{u}{v}\right)$

b) $\dfrac{d}{dx}(uv)$

c) $\dfrac{d}{dx}(5u-2v+4uv)$

d) $\dfrac{d}{dx}\left(\dfrac{v}{u}\right)$

8. Solve for a and b in order for $f(x)$ to be both continuous and differentiable at $x=1$. (be sure to use the definition of continuity)

$$f(x)=\begin{cases} x^2+2 & ;x\le 1 \\ a\left(x-\frac{1}{x}\right)+b & ;x>1 \end{cases}$$

9. For each of the following, find the equation of the tangent line to the given function at the indicated point.

a) $f(x)=(x^3-3x+1)(x+2)$ at the point (1, –3).

b) $y=\dfrac{8}{4+x^2}$ at the point (–2, 1).

10. At what point on the graph of $y=\frac{1}{2}x^2$ is the tangent line parallel to the line $2x-4y=3$?

A) $\left(\frac{1}{2},\frac{1}{2}\right)$
B) $\left(\frac{1}{2},\frac{1}{8}\right)$
C) $\left(1,-\frac{1}{4}\right)$
D) $\left(1,\frac{1}{2}\right)$
E) $(2,2)$

11. Let f be a differentiable function such that $f(3)=2$ and $f'(3)=5$. If the tangent line to the graph of f at $x=3$ is used to find an approximation to a zero of f, that approximation is

A) 0.4
B) 0.5
C) 2.6
D) 3.4
E) 5.5

12. An equation of the line tangent to the graph of $y=\dfrac{2x+3}{3x-2}$ at the point (1, 5) is

A) $13x-y=8$
B) $13x+y=18$
C) $x-13y=64$
D) $x+13y=66$
E) $-2x+3y=13$

13. What is the instantaneous rate of change at $x = 2$ of the function f given by $f(x) = \dfrac{x^2 - 2}{x - 1}$?

A) -2
B) $\frac{1}{6}$
C) $\frac{1}{2}$
D) 2
E) 6

14. If u, v, and w are nonzero differentiable functions of x, then the $\dfrac{d}{dx}\left(\dfrac{uv}{w}\right)$ is

A) $\dfrac{uv' + u'v}{w'}$

B) $\dfrac{u'v'w - uvw'}{w^2}$

C) $\dfrac{uvw' - uv'w - u'vw}{w^2}$

D) $\dfrac{u'vw + uv'w + uvw'}{w^2}$

E) $\dfrac{uv'w + u'vw - uvw'}{w^2}$

15. When an object is thrown off a 100 foot cliff with an initial velocity of 40 feet/second, the height h, in feet, of the object can be modeled as a function of time t, in seconds, using the function

$$h(t) = -16t^2 + 45t + 100.$$

a) Find $\dfrac{dh}{dt}$ … What is the unit of measurement for this equation?

b) Find $\dfrac{d^2h}{dt^2}$ … What is the unit of measurement for this equation?

16. Let $g(x) = x - \dfrac{1}{x}$. Find the following:

a) $g'(x)$ b) $g''(x)$ c) The tangent line equation when $x = 2$