AP Calculus
3.2 Worksheet
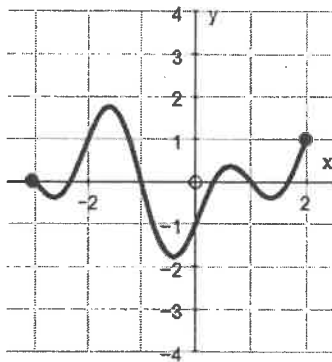
*All work must be shown in this course for full credit. Unsupported answers may receive NO credit.*
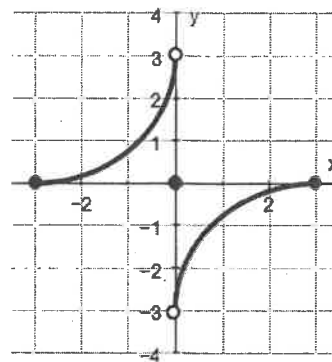
1. When does a derivative fail to exist?

For questions 2 – 4, the graph of a function over a closed interval $D$ is given. At what domain points does the function appear to be
      a) differentiable?     b) continuous but not differentiable     c) neither continuous or differentiable?
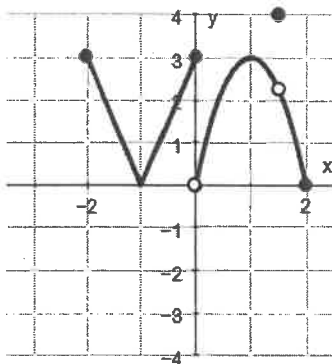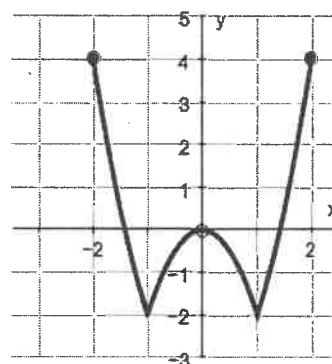
2.   $D: \ -3 \le x \le 2$                3.   $D: \ -3 \le x \le 3$



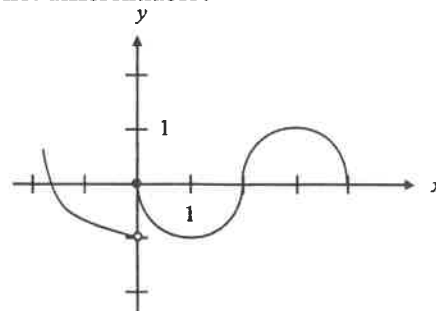4.   $D: \ -2 \le x \le 2$                5.   $D: \ -2 \le x \le 2$



6. The graph of the function $f$ shown in the figure below has a vertical tangent at the point $(2, 0)$ and horizontal tangents at the points $(1, -1)$ and $(3, 1)$. For what values of $x$, $-2 < x < 4$, is $f$ not differentiable?

    A)     0 only

    B)     0 and 2 only

    C)     1 and 3 only

    D)     0, 1, and 3 only

    E)     0, 1, 2, and 3

7. Suppose $f(x) = 2 + |x+3|$ .

    a) What is the value of $f'(3)$? Explain your answer.

    b) What is the value of $f'(-3)$? Explain your answer.

8. What are the three different derivative "formulas" ?... (don't forget to use a limit)

9. If $f$ is a function such that $\lim\limits_{x \to 2} \dfrac{f(x) - f(2)}{x - 2} = 0$, which of the following must be true?

    A)    The limit of $f(x)$ as $x$ approaches 2 does not exist.
    B)    $f$ is not defined at $x = 2$.
    C)    The derivative of $f$ at $x = 2$ is 0.
    D)    $f$ is continuous at $x = 0$.
    E)    $f(2) = 0$

10. Let $f$ be a function such that $\lim\limits_{h \to 0} \dfrac{f(2+h) - f(2)}{h} = 5$. Which of the following must be true?

    I. $f$ is continuous at $x = 2$.
    II. $f$ is differentiable at $x = 2$.
    III. The derivative of $f$ is continuous at $x = 2$.

    A) I only      B) II only      C) I and II only      D) I and III only      E) II and III only

11. Let $f$ be a function that is differentiable on the open interval $(0, 10)$. If $f(2) = -5$, and $f(5) = 5$, and $f(9) = -5$, each of the following statements MUST be true. Explain why each statement must be true.
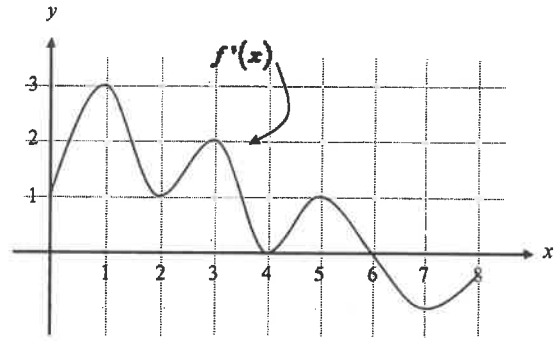
    a) $f$ has at least 2 zeros.

    b) The graph of $f$ has at least one horizontal tangent line.

    c) For some $c$, $2 < c < 5$, $f(c) = 3$.

12. The function $f$ is defined on the closed interval $[0, 8]$. The graph of its derivative $f'$ is shown below. The point $(3, 5)$ is on the graph of $f(x)$. An equation of the tangent line to the graph of $f$ at $(3, 5)$ is

A)  $y = 2$

B)  $y = 5$

C)  $y - 5 = 2(x - 3)$

D)  $y + 5 = 2(x - 3)$

E)  $y + 5 = 2(x + 3)$



13. Let $g(x) = \begin{cases} 3x - 2 & \text{if } x \le 0 \\ x^2 - 1 & \text{if } x > 0 \end{cases}$. Which of the following is equal to the left-hand derivative of $g$ at $x = 0$?

A) $2x$      B) $3$      C) $0$      D) $\infty$      E) $-\infty$

14. Suppose $f(x) = \begin{cases} 3 - x & \text{if } x < 1 \\ mx^2 + nx & \text{if } x \ge 1 \end{cases}$

a) If the function is continuous, what is the relationship between $m$ and $n$. (Use the definition of continuity!)

b) What is the derivative of the portion of the graph where $x < 1$.

c) Using whatever method you wish to show/explain, find the derivative of the portion of the graph where $x \ge 1$.

d) In order for $f(x)$ to be differentiable at $x = 1$, what is the relationship between the answers in part $b$ and $c$?

e) Using your answers from part (d) and from part (a), solve for $m$ and $n$.

One of the four (4) required calculator skills on the AP exam is for you to take a derivative at a point. Use your calculator to answer the following questions. Be sure to use correct mathematical notation.

15. Using your calculator, find the equation of the tangent line to the graph of $f(x) = x^3 + x^2$ when $x = 2$. Show your work using correct notation.

16. When an object falls its distance traveled (in meters) can be modeled by the equation $h(t) = 4.9t^2$. The derivative of $h$ with respect to $t$ is the velocity of the object. Find the velocity of the object at $t = 3$ seconds.

17. Suppose $f(x) = |4 - x^2|$.

    a) Find the slope of the function when $x = 3$.

    b) Find the slope of the function when $x = 2$.

    c) Graph the function and explain any issues with your answer from part (b).