Aim: How can we determine differentiability?

Get Ready: Please determine if the function is continuous. If not, state the domain where the function is not continuous.

1. $f(x)=\begin{cases}3x+2, x\le -2\\ x^2-8, x>-2\end{cases}$

2. $g(x)=\sqrt{x^2+2x-8}$

3. $g(x)=\sqrt[5]{3x-5}$

4. $h(x)=\dfrac{x+2}{x^2-x-6}$

5. $f(x)=\begin{cases}2x^2-4, x>3\\ 3x+4, x\le 3\end{cases}$

6. Find "a" that makes f(x) continuous

$f(x)=\begin{cases}a^2x^2+3x+2, x<1\\ -3ax+9, x\ge 1\end{cases}$

I. Differentiability:

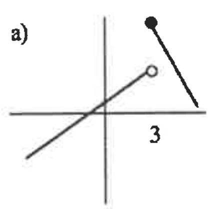
Differentiability At a Point: Function *f(x)* is differentiable at *x=c* if $f'(c)$ exists.($f'(c)$ =real number)

Differentiability On An Interval: Function *f(x)* is differentiable on an interval *(a,b)* if and only if it is differentiable for every value of *x* on the interval *(a,b)*.

Differentiability: Function *f(x)* is differentiable if and only if it is differentiable at every value of *x*.

II. Differentiability→"Smooth" A differentiable curve will have no sharp points(cusps or corners) or vertical tangent lines. A curve must be continuous at all points. Differentiability implies continuity. Continuity does not imply differentiability.

III. Determine whether the following functions are continuous, differentiable, or both.

a)


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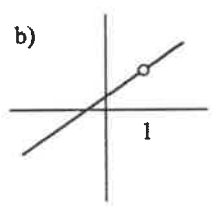

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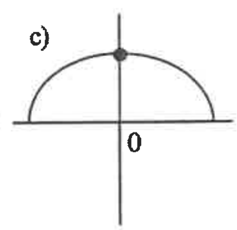

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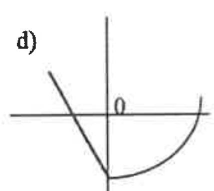

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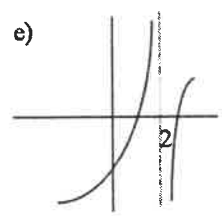

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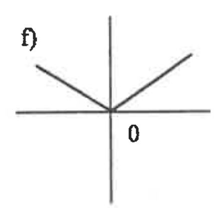
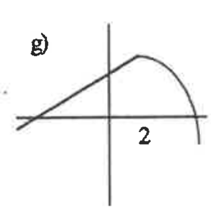


g)


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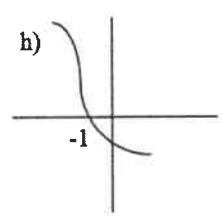

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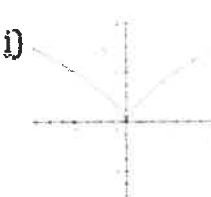

Aim: How can we determine differentiability?

IV. Determine whether the functions are differentiable.

a. $f(x) = x^2 - 6x + 1$

b. $f(x) = \dfrac{x^2 - x - 12}{x + 3}$

c. $f(x) = \sin x$

d. $f(x) = \dfrac{\sin x}{x}$

V. Determine if the function if differentiability at the point where the rule(curve) changes.

a. $f(x) = \begin{cases} 4 - x, x < 2 \\ x^2 - 6x + 10, x \geq 2 \end{cases}$

b. $f(x) = \begin{cases} -x - 4, x < -1 \\ x^2 + x - 3, x \geq -1 \end{cases}$

c. $f(x) = \begin{cases} 4 - \sqrt[3]{x - 4}, x < 4 \\ \sqrt{x + 5}, x \geq 4 \end{cases}$

d. $f(x) = \begin{cases} \sin x, x \geq 0 \\ x - 3x^2, x < 0 \end{cases}$

VI. Find the values of "a" and "b" that make *f(x)* differentiable.

a. $f(x) = \begin{cases} ax^2 + 1, x \geq 1 \\ bx - 3, x < 1 \end{cases}$

b. $f(x) = \begin{cases} ax^3 + 1, x < 2 \\ b(x - 3)^2 + 10, x \geq 2 \end{cases}$

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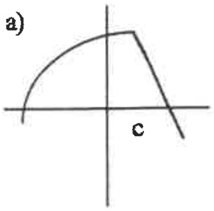
VII. Mixed Practice

1. Determine whether the following functions are continuous, differentiable, both, or neither at $x = c$.

a)



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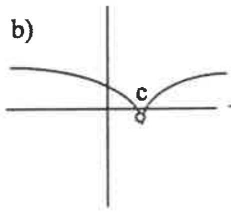

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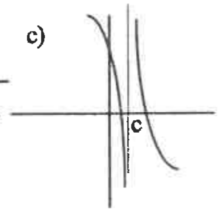
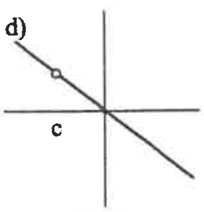
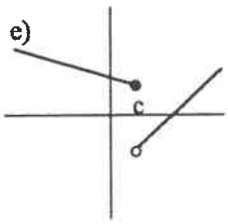


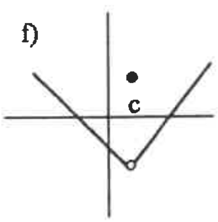
d)



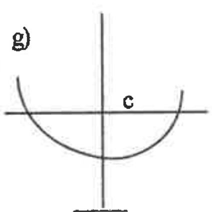
e)



f)



g)



2. Determine whether the following functions are continuous, differentiable, both, or neither. Show necessary work.

a. $f(x) = \begin{cases} x^2, x \geq 0 \\ x, x < 0 \end{cases}$

b. $f(x) = \begin{cases} x^2 + 1, x \geq 0 \\ x^3 + 1, x < 0 \end{cases}$

c. $f(x) = \begin{cases} 4 - x^2, x < 1 \\ 2x + 2, x \geq 1 \end{cases}$

d. $f(x) = \begin{cases} x^2 + x - 7, x \geq 2 \\ 5x - 11, x < 2 \end{cases}$

e. $f(x) = \begin{cases} \sqrt{x} - 3, x > 1 \\ \frac{1}{2}x - \frac{5}{2}, x \leq 1 \end{cases}$

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f. $f(x)=\begin{cases}\sin(x), x>0\\ x, x\le 0\end{cases}$

g. $f(x)=\begin{cases}\cos(x), x\ge 0\\ 1-x^2, x<0\end{cases}$

h. $f(x)=\begin{cases}3+(x+2)^{\frac{1}{3}}, x\ge -2\\ 3-(x+2)^{\frac{2}{3}}, x<-2\end{cases}$

3. Find the values of "a" and "b" that make the function differentiable.

a. $f(x)=\begin{cases}x^3, x\ge 1\\ a(x-2)^2+b, x<1\end{cases}$

b. $f(x)=\begin{cases}ax^2+10, x\ge 2\\ x^2-6x+b, x<2\end{cases}$

c. $f(x)=\begin{cases}\frac{a}{x}, x\ge 1\\ 12-bx^2, x<1\end{cases}$