Section 6.4 Exponential Growth and Decay

<u>Law of Exponential Change</u>

If y changes at a rate proportional to the amount present (that is $\frac{dy}{dt} = ky$) and $y=y_0$ when t=0 then...

Now try exercise #11

<u>Continuously Compounded Interest</u>

For interest compounded daily, weekly or monthly we have this formula from previous courses:

$A(t) = A_0(1+\frac{r}{k})^{kt}$ k=12 monthly k=52 weekly k=365 daily

Interest paid according to the following formula is said to be compounded continuously. The #r is the continuous interest rate. A_0 is the original investment.

$$\frac{dA}{dt} = rA \qquad A(0) = A_0 \qquad A(t) = A_0e^{rt}$$

Ex: Suppose you deposit $800 in an account that pays 6.3% annual interest. How much will you have 8 years later if interest is (a) compounded continuously and (b) compounded quarterly?

Now try exercise #19

<u>Radioactivity</u>

Radioactive decay: $\frac{dy}{dt} = -ky, k > 0 \qquad y(0) = y_0 \qquad y = y_0e^{-kt}\ k > 0$

Half life is the time required for half of the radioactive nuclei present in a sample to decay

We can derive it here:

Now try exercise #21

Modeling Growth with other Bases

Exponential growth can be modeled with any positive base not equal to 1, enabling us to choose a conventional base to fit a given growth pattern. Please do exploration on page 353.

(It is important to note that only when b=e that k is in the exponent. In general, the coefficient of t is the reciprocal of the time period required for the population to grow by a factor of b.)

Ex: At the beginning of the summer, the population of a hive of wasps is growing at a rate proportional to the population. From a population of 10 on May 1, the number of wasps grows to 50 in 30 days. If the growth continues to follow the same model, how many days after May 1 will the population reach 100?

Now try exercise #23

Ex: Scientists who use Carbon-14 dating use 5700 years for its half life. Find the age of a sample in which 10% of the radioactive nuclei originally present have decayed.

Now try exercise #25

Newton's Law of Cooling

T is the temperature of an object at time t and T_s is the surrounding temperature.

$$\frac{dT}{dt} = -k(T - T_s) \qquad T - T_s = (T_0 - T_s)e^{-kt}$$

Ex: A hardboiled egg at 98° is put in a pan under 18° running water to cool. After 5 minutes, the eggs temperature is 38°. How much longer until it reaches 20°?

Now try exercise #31