10.4 Modeling Planar Motion-Apps of Vectors

Definitions: Velocity, Speed, Acceleration & Direction of Motion

Suppose a particle moves along a smooth curve in the plane so that its position at any time t is $s(t) = (x(t), y(t))$.

1. Position Vector: $r(t) = \langle x(t), y(t) \rangle$
2. Velocity Vector: $v(t) = \langle \frac{dx}{dt}, \frac{dy}{dt} \rangle$
3. Speed-the magnitude of $v(|v|)$: $\sqrt{(\frac{dx}{dt})^2 + (\frac{dy}{dt})^2}$ (must have a point-this is a number)
4. Acceleration Vector: $a(t) = \langle \frac{d^2x}{dt^2}, \frac{d^2y}{dt^2} \rangle$
5. Direction of Motion is the direction vector: $\frac{v}{|v|}$

Definitions: Displacement and Distance Traveled

Suppose a particle moves along a path in the plane so that its velocity at any given time t is $v(t) = (v_1(t), v_2(t))$

1. Displacement from t=a to t=b: $\langle \int_a^b v_1(t), \int_a^b v_2(t) \rangle$
2. Position at time t=#: $\langle inital\ condition\ vector \rangle + \langle displacement\ vector \rangle$
3. Distance Traveled from t=a to t=b: $\int_a^b |v(t)| dt = \int_a^b \sqrt{(v_1(t))^2 + (v_2(t))^2} dt$

Ex: A particle moves in the plane so that its position at any time $t \geq 0$ is given by $(\sin(t), \frac{t^2}{2})$.

a. Find the position vector:

b. Find the velocity vector:

c. Find the acceleration vector:

d. Describe the position and motion of the particle at t=6.

Ex: A particle moves in the plane with position vector $r(t) = \langle \sin(3t), \cos(5t) \rangle$.

a. Find the velocity vector:

b. Find the acceleration vector:

c. Determine the path of the particle:

Ex: A particle moves in the plane with position vector $\langle \cos(3t), \sin(2t) \rangle$.

a. Find the velocity vector:

b. Find the acceleration vector:

c. Determine the path of the particle:

Ex: A particle moves in an elliptical path so that its position at any time $t \geq 0$ is given by (4sint, 2cost).

a. Find the velocity and acceleration vectors:

b. Find $v(\frac{\pi}{4})$, $a(\frac{\pi}{4})$, speed at $\frac{\pi}{4}$ and direction of motion at $\frac{\pi}{4}$

c. Sketch the path of the particle:

d. Does it travel clockwise or counterclockwise?

e. Distance traveled from t=0 to $t=\frac{\pi}{4}$

Ex: A particle moves in the plane $t \geq 0$ with position $x=\sin 4t \cos t$, $y=\sin 2t$.

a. Find the velocity at $t=\frac{5\pi}{4}$

b. Find the speed at $t=\frac{5\pi}{4}$

c. Find the direction of motion at $t=\frac{5\pi}{4}$

d. Draw the path of the particle:

e. Find the distance traveled from t=0 to $t=\frac{5\pi}{4}$

Ex: A particle moves with velocity vector $v(t) = \langle \pi - 3\pi \cos(\pi t), 2t - \pi \sin(\pi t) \rangle$. At t=0 the particle is at (1,5).

a. Find the position at t=4.

b. What is the total distance traveled from t=0 to t=4.

c. Determine the path the particle travels going from (1,5) to (9,21):