

3. The graphs of the polar curves $r = 2$ and $r = 3 + 2\cos \theta$ are shown in the figure above. The curves intersect when $\theta = \frac{2\pi}{3}$ and $\theta = \frac{4\pi}{3}$.
- (a) Let R be the region that is inside the graph of $r = 2$ and also inside the graph of $r = 3 + 2\cos \theta$, as shaded in the figure above. Find the area of R .
- (b) A particle moving with nonzero velocity along the polar curve given by $r = 3 + 2\cos \theta$ has position $(x(t), y(t))$ at time t , with $\theta = 0$ when $t = 0$. This particle moves along the curve so that $\frac{dr}{dt} = \frac{dr}{d\theta}$. Find the value of $\frac{dr}{dt}$ at $\theta = \frac{\pi}{3}$ and interpret your answer in terms of the motion of the particle.
- (c) For the particle described in part (b), $\frac{dy}{dt} = \frac{dy}{d\theta}$. Find the value of $\frac{dy}{dt}$ at $\theta = \frac{\pi}{3}$ and interpret your answer in terms of the motion of the particle.

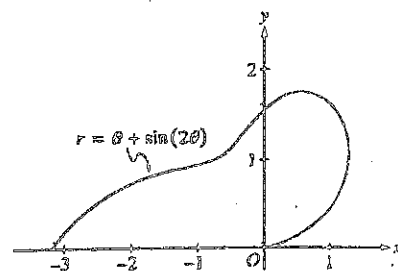
3.5

AP Calculus BC-2

Final Draft for Scoring

2005

The curve above is drawn in the xy -plane and is described by the equation in polar coordinates $r = \theta + \sin(2\theta)$ for $0 \leq \theta \leq \pi$, where r is measured in meters and θ is measured in radians. The derivative of r with respect to θ is given by $\frac{dr}{d\theta} = 1 + 2\cos(2\theta)$.



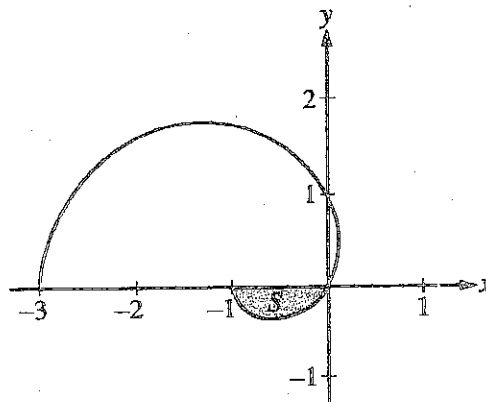
- (a) Find the area bounded by the curve and the x -axis.
- (b) Find the angle θ that corresponds to the point on the curve with x -coordinate -2 .
- (c) For $\frac{\pi}{3} < \theta < \frac{2\pi}{3}$, $\frac{dr}{d\theta}$ is negative. What does this fact say about r ? What does this fact say about the curve?
- (d) Find the value of θ in the interval $0 \leq \theta \leq \frac{\pi}{2}$ that corresponds to the point on the curve in the first quadrant with greatest distance from the origin. Justify your answer.

CALCULUS BC
SECTION II, Part B

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.



4. The graph of the polar curve $r = 1 - 2\cos \theta$ for $0 \leq \theta \leq \pi$ is shown above. Let S be the shaded region in the third quadrant bounded by the curve and the x -axis.
- (a) Write an integral expression for the area of S .
- (b) Write expressions for $\frac{dx}{d\theta}$ and $\frac{dy}{d\theta}$ in terms of θ .
- (c) Write an equation in terms of x and y for the line tangent to the graph of the polar curve at the point where $\theta = \frac{\pi}{2}$. Show the computations that lead to your answer.
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