10.2 Vectors in the Plane

A **two-dimensional vector** v is an ordered pair of real numbers, denoted in **component form** as $\langle a, b\rangle$. The numbers a and b are the **components** of the vector v. The **standard representation** of the vector $\langle a, b\rangle$ is the arrow from the origin to the point (a,b). The **magnitude** (or absolute value) of v, denoted $|v|$, is the length of the arrow, and the direction of v is the direction in which the arrow is pointing. The vector 0= $\langle 0,0\rangle$ called the **zero vector**, has zero length and no direction.

The **magnitude** of the vector $\langle a, b\rangle$ is the nonnegative real number $|\langle a, b\rangle| = \sqrt{a^2 + b^2}$. The direction angle of a nonzero vector is the smallest nonnegative angle θ formed with the positive x axis as the initial ray and the standard representation of v as the terminal ray.

<u>HMT (Head Minus Tail) Rule</u> – If any arrow has initial point (x_1, y_1) and terminal point (x_2, y_2), it represents the vector $\langle x_2 - x_1, y_2 - y_1\rangle$.

<u>Find Magnitude and Direction</u>

<u>Find Component Form and Magnitude</u>

1. Find the component of a vector with magnitude 6 and direction angle $\frac{3\pi}{2}$.

2. Find the component form and magnitude of a vector from (4,-7) to (-1,5).

Vector Operations

Let u=$\langle u_1 + u_2 \rangle$ and v=$\langle v_1 + v_2 \rangle$

The **sum or resultant** of the vectors u and v is $u + v = \langle u_1 + v_1, u_2 + v_2 \rangle$.

The **product of the scalar k and the vector** u is $ku = k\langle u_1, u_2 \rangle = \langle ku_1, ku_2 \rangle$.

The **opposite** of a vector v is $-v = (-1)v$ so $u - v = u + (-v)$.

$\frac{v}{|v|}$ is a vector of magnitude 1, called a unit vector. Its component form is $\langle cos\theta, sin\theta \rangle$, where θ is the direction angle of **v**. For this reason $\frac{v}{|v|}$ is sometimes called the **direction vector** of **v**.

The sum of two vectors **u** and **v** can be represented geometrically by arrows in two ways:

Performing Operations on Vectors

Properties of Vectors

1. u+v=v+u
2. (u+v)+w=u+(v+w)
3. u+0=u
4. u+(-u)=0
5. 0(u)=0
6. 1u=u
7. a(bu)=(ab)u
8. a(u+v)=au+av
9. (a+b)u=au+bu