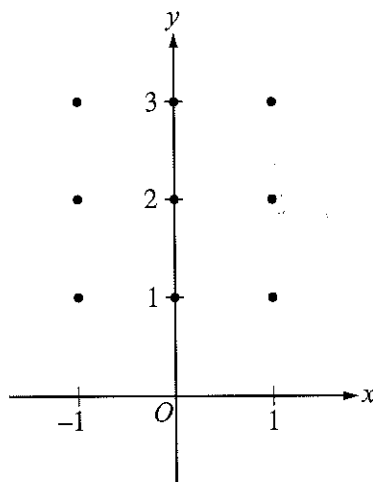


1998 Calculus BC Free-Response Questions

4. Consider the differential equation given by $\frac{dy}{dx} = \frac{xy}{2}$.
- (a) On the axes provided below, sketch a slope field for the given differential equation at the nine points indicated.



- (b) Let $y = f(x)$ be the particular solution to the given differential equation with the initial condition $f(0) = 3$. Use Euler's method starting at $x = 0$, with a step size of 0.1, to approximate $f(0.2)$. Show the work that leads to your answer.
- (c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(0) = 3$. Use your solution to find $f(0.2)$.
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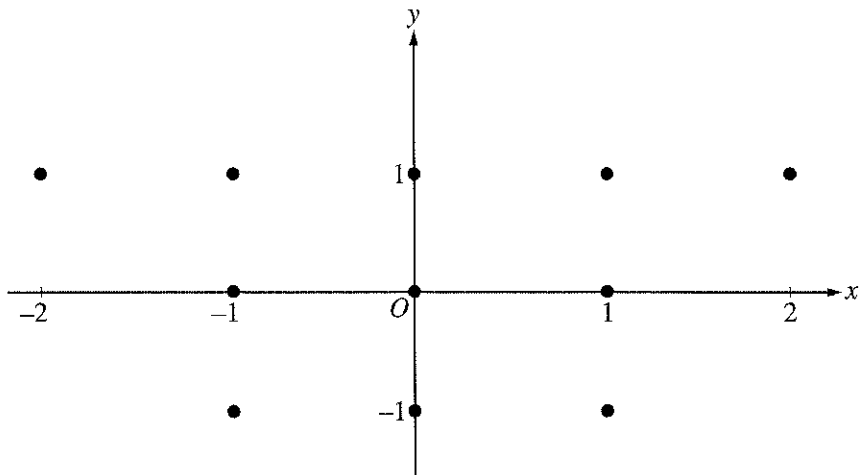
GO ON TO THE NEXT PAGE

2000 AP[®] CALCULUS BC FREE-RESPONSE QUESTIONS

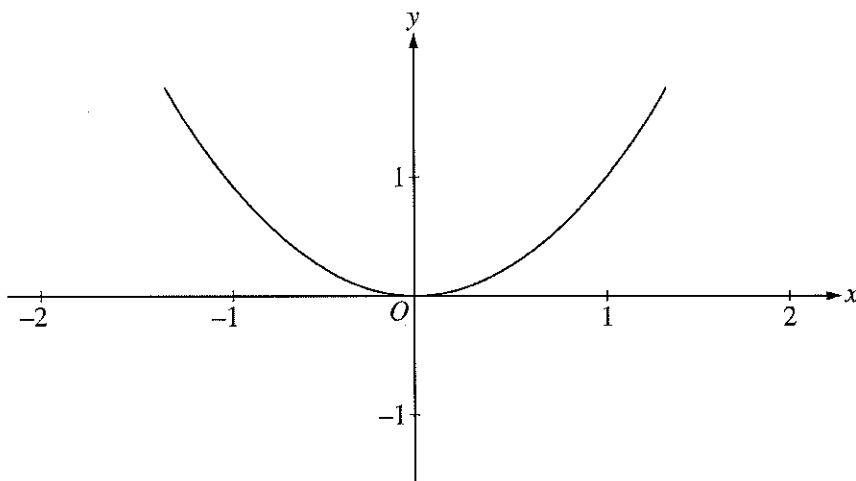
6. Consider the differential equation given by $\frac{dy}{dx} = x(y - 1)^2$.

(a) On the axes provided, sketch a slope field for the given differential equation at the eleven points indicated.

(Note: Use the axes provided in the pink test booklet.)



(b) Use the slope field for the given differential equation to explain why a solution could not have the graph shown below.



(c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(0) = -1$.

(d) Find the range of the solution found in part (c).

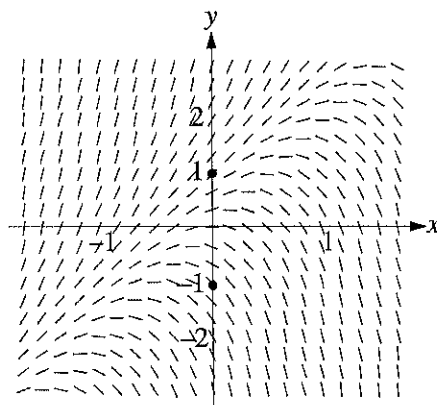
END OF EXAMINATION

2002 AP[®] CALCULUS BC FREE-RESPONSE QUESTIONS

5. Consider the differential equation $\frac{dy}{dx} = 2y - 4x$.

- (a) The slope field for the given differential equation is provided. Sketch the solution curve that passes through the point $(0, 1)$ and sketch the solution curve that passes through the point $(0, -1)$.

(Note: Use the slope field provided in the pink test booklet.)



- (b) Let f be the function that satisfies the given differential equation with the initial condition $f(0) = 1$. Use Euler's method, starting at $x = 0$ with a step size of 0.1, to approximate $f(0.2)$. Show the work that leads to your answer.
- (c) Find the value of b for which $y = 2x + b$ is a solution to the given differential equation. Justify your answer.
- (d) Let g be the function that satisfies the given differential equation with the initial condition $g(0) = 0$. Does the graph of g have a local extremum at the point $(0, 0)$? If so, is the point a local maximum or a local minimum? Justify your answer.

6. The Maclaurin series for the function f is given by

$$f(x) = \sum_{n=0}^{\infty} \frac{(2x)^{n+1}}{n+1} = 2x + \frac{4x^2}{2} + \frac{8x^3}{3} + \frac{16x^4}{4} + \cdots + \frac{(2x)^{n+1}}{n+1} + \cdots$$

on its interval of convergence.

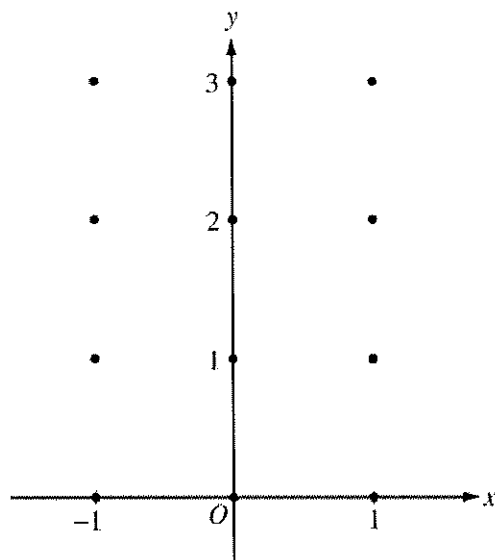
- (a) Find the interval of convergence of the Maclaurin series for f . Justify your answer.
- (b) Find the first four terms and the general term for the Maclaurin series for $f'(x)$.
- (c) Use the Maclaurin series you found in part (b) to find the value of $f'\left(-\frac{1}{3}\right)$.

END OF EXAMINATION

2004 AP[®] CALCULUS AB FREE-RESPONSE QUESTIONS

6. Consider the differential equation $\frac{dy}{dx} = x^2(y - 1)$.

- (a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated.
(Note: Use the axes provided in the pink test booklet.)



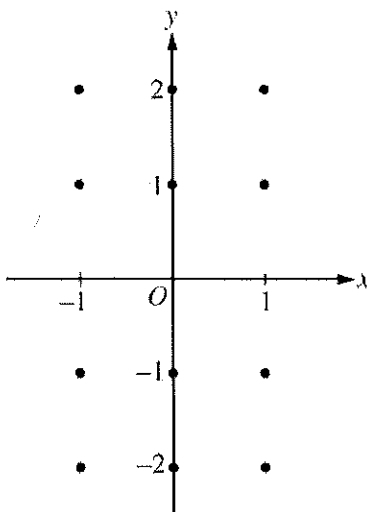
- (b) While the slope field in part (a) is drawn at only twelve points, it is defined at every point in the xy -plane. Describe all points in the xy -plane for which the slopes are positive.
- (c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(0) = 3$.
-

END OF EXAMINATION

2005 AP[®] CALCULUS AB FREE-RESPONSE QUESTIONS

6. Consider the differential equation $\frac{dy}{dx} = -\frac{2x}{y}$.

- (a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated.
(Note: Use the axes provided in the pink test booklet.)



- (b) Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(1) = -1$.
Write an equation for the line tangent to the graph of f at $(1, -1)$ and use it to approximate $f(1.1)$.
- (c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(1) = -1$.
-

WRITE ALL WORK IN THE TEST BOOKLET.

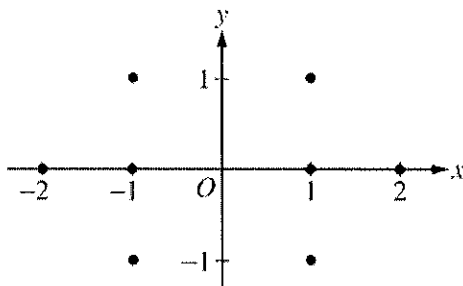
END OF EXAM

2006 AP[®] CALCULUS AB FREE-RESPONSE QUESTIONS

5. Consider the differential equation $\frac{dy}{dx} = \frac{1+y}{x}$, where $x \neq 0$.

(a) On the axes provided, sketch a slope field for the given differential equation at the eight points indicated.

(Note: Use the axes provided in the pink exam booklet.)



(b) Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(-1) = 1$ and state its domain.

6. The twice-differentiable function f is defined for all real numbers and satisfies the following conditions:

$$f(0) = 2, \quad f'(0) = -4, \quad \text{and} \quad f''(0) = 3.$$

(a) The function g is given by $g(x) = e^{ax} + f(x)$ for all real numbers, where a is a constant. Find $g'(0)$ and $g''(0)$ in terms of a . Show the work that leads to your answers.

(b) The function h is given by $h(x) = \cos(kx)f(x)$ for all real numbers, where k is a constant. Find $h'(x)$ and write an equation for the line tangent to the graph of h at $x = 0$.

WRITE ALL WORK IN THE PINK EXAM BOOKLET.

END OF EXAM