Quick Review 8.1 *(For help, go to Sections 1.2, 2.1, and 2.2.)*

In Exercises 1 and 2, let $f(x) = \dfrac{x}{x+3}$. Find the values of f.

1. $f(5)$
2. $f(-2)$

In Exercises 3 and 4, evaluate the expression $a + (n-1)d$ for the given values of a, n, and d.

3. $a = -2, n = 3, d = 1.5$
4. $a = -7, n = 5, d = 3$

In Exercises 5 and 6, evaluate the expression ar^{n-1} for the given values of a, r, and n.

5. $a = 1.5, r = 2, n = 4$
6. $a = -2, r = 1.5, n = 3$

In Exercises 7–10, find the value of the limit.

7. $\lim\limits_{x\to\infty} \dfrac{5x^3 + 2x^2}{3x^4 + 16x^2}$
8. $\lim\limits_{x\to 0} \dfrac{\sin(3x)}{x}$
9. $\lim\limits_{x\to\infty} \left(x \sin\left(\dfrac{1}{x}\right)\right)$
10. $\lim\limits_{x\to\infty} \dfrac{2x^3 + x^2}{x+1}$

Section 8.1 Exercises

In Exercises 1–4, find the first six terms and the 50th term of the sequence with specified nth term.

1. $a_n = \dfrac{n}{n+1}$
2. $b_n = 3 - \dfrac{1}{n}$
3. $c_n = \left(1 + \dfrac{1}{n}\right)^n$
4. $d_n = n^2 - 3n$

In Exercises 5–10, find the first four terms and the eighth term of the recursively defined sequence.

5. $a_1 = 3, a_n = a_{n-1} - 2$ for all $n \geq 2$
6. $b_1 = -2, b_n = b_{n-1} + 1$ for all $n \geq 2$
7. $c_1 = 2, c_n = 2c_{n-1}$ for all $n \geq 2$
8. $d_1 = 10, d_n = 1.1d_{n-1}$ for all $n \geq 2$
9. $u_1 = 1, u_2 = 1, u_n = u_{n-1} + u_{n-2}$ for all $n \geq 3$
10. $v_1 = -3, v_2 = 2, v_n = v_{n-1} + v_{n-2}$ for all $n \geq 3$

In Exercises 11–14, the sequences are arithmetic. Find

(a) the common difference,

(b) the eighth term,

(c) a recursive rule for the nth term, and

(d) an explicit rule for the nth term.

11. $-2, 1, 4, 7, \ldots$
12. $15, 13, 11, 9, \ldots$
13. $1, 3/2, 2, 5/2, \ldots$
14. $3, 3.1, 3.2, 3.3, \ldots$

In Exercises 15–18, the sequences are geometric. Find

(a) the common ratio,

(b) the ninth term,

(c) a recursive rule for the nth term, and

(d) an explicit rule for the nth term.

15. $8, 4, 2, 1, \ldots$
16. $1, 1.5, 2.25, 3.375, \ldots$
17. $-3, 9, -27, 81, \ldots$
18. $5, -5, 5, -5, \ldots$
19. The second and fifth terms of an arithmetic sequence are -2 and 7, respectively. Find the first term and a recursive rule for the nth term.
20. The fifth and ninth terms of an arithmetic sequence are 5 and -3, respectively. Find the first term and an explicit rule for the nth term.
21. The fourth and seventh terms of a geometric sequence are 3010 and 3,010,000, respectively. Find the first term, common ratio, and an explicit rule for the nth term.
22. The second and seventh terms of a geometric sequence are $-1/2$ and 16, respectively. Find the first term, common ratio, and an explicit rule for the nth term.

In Exercises 23–30, draw a graph of the sequence $\{a_n\}$.

23. $a_n = \dfrac{n}{n^2+1}, \quad n = 1, 2, 3, \ldots$
24. $a_n = \dfrac{n-2}{n+2}, \quad n = 1, 2, 3, \ldots$
25. $a_n = (-1)^n \dfrac{2n+1}{n}, \quad n = 1, 2, 3, \ldots$
26. $a_n = \left(1 + \dfrac{2}{n}\right)^n, n = 1, 2, 3, \ldots$
27. $u_1 = 2, \quad u_n = 3u_{n-1}$ for all $n \geq 2$
28. $u_1 = 2, \quad u_n = u_{n-1} + 3$ for all $n \geq 2$
29. $u_1 = 3, \quad u_n = 5 - \dfrac{1}{2}u_{n-1}$ for all $n \geq 2$
30. $u_1 = 5, \quad u_n = u_{n-1} - 2$ for all $n \geq 2$

In Exercises 31–40, determine the convergence or divergence of the sequence with given nth term. If the sequence converges, find its limit.

31. $a_n = \dfrac{3n+1}{n}$
32. $a_n = \dfrac{2n}{n+3}$
33. $a_n = \dfrac{2n^2 - n - 1}{5n^2 + n + 2}$
34. $a_n = \dfrac{n}{n^2+1}$
35. $a_n = (-1)^n \dfrac{n-1}{n+3}$
36. $a_n = (-1)^n \dfrac{n+1}{n^2+2}$
37. $a_n = (1.1)^n$
38. $a_n = (0.9)^n$
39. $a_n = n \sin\left(\dfrac{1}{n}\right)$
40. $a_n = \cos\left(n\dfrac{\pi}{2}\right)$

In Exercises 41–44, use the Sandwich Theorem to show that the sequence with given nth term converges and find its limit.

41. $a_n = \dfrac{\sin n}{n}$
42. $a_n = \dfrac{1}{2^n}$
43. $a_n = \dfrac{1}{n!}$
44. $a_n = \dfrac{\sin^2 n}{2^n}$

In Exercises 45–48, match the graph or table with the sequence with given nth term.

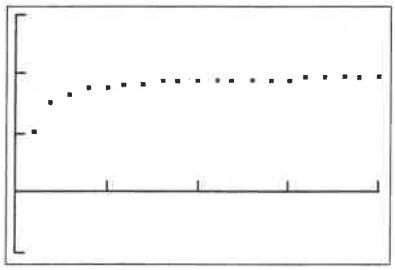
45. $a_n = \dfrac{2n-1}{n}$ **46.** $b_n = (-1)^n \dfrac{3n+1}{n+3}$

47. $c_n = \dfrac{n+1}{n}$ **48.** $d_n = \dfrac{4}{n+2}$

n	u(n)
1	1.3333
2	1
3	.8
4	.66667
5	.57143
6	.5
7	.44444

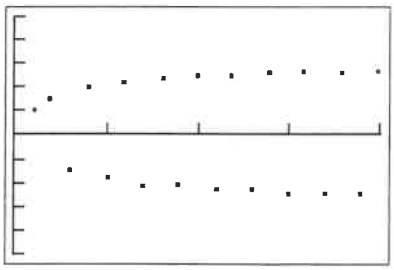
n = 1

(a)



[0, 20] by [–1, 3]

(b)



[0, 20] by [–5, 5]

(c)

n	u(n)
1	2
2	1.5
3	1.3333
4	1.25
5	1.2
6	1.1667
7	1.1429

n = 1

(d)

Standardized Test Questions

You should solve the following problems without using a graphing calculator.

49. True or False If the first two terms of an arithmetic sequence are negative, then all its terms are negative. Justify your answer.

50. True or False If the first two terms of a geometric sequence are positive, then all its terms are positive. Justify your answer.

51. Multiple Choice The first and third terms of an arithmetic sequence are -1 and 5, respectively. Which of the following is the sixth term?

(A) -25 (B) 11 (C) 14 (D) 29 (E) 3125

52. Multiple Choice The second and third terms of a geometric sequence are 2.5 and 1.25, respectively. Which of the following is the first term?

(A) -5 (B) -2.5 (C) 0.625 (D) 3.75 (E) 5

53. Multiple Choice Which of the following is the limit of the sequence with nth term $a_n = n \sin\left(\dfrac{3\pi}{n}\right)$?

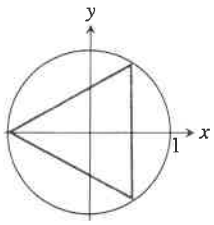
(A) 1 (B) π (C) 2π (D) 3π (E) 4π

54. Multiple Choice Which of the following is the limit of the sequence with nth term $a_n = (-1)^n \dfrac{3n-1}{n+2}$?

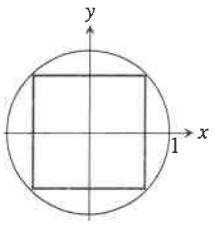
(A) -3 (B) 0 (C) 2 (D) 3 (E) diverges

Explorations

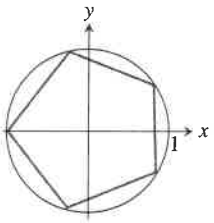
55. *Connecting Geometry and Sequences* In the sequence of diagrams that follow, regular polygons are inscribed in unit circles with at least one side of each polygon perpendicular to the x-axis.



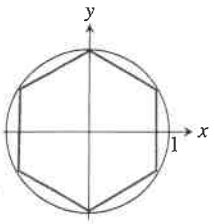
(a)



(b)



(c)



(d)

(a) Prove that the perimeter of each polygon in the sequence is given by $a_n = 2n \sin(\pi/n)$, where n is the number of sides in the polygon.

(b) Determine $\lim_{n\to\infty} a_n$.