Quick Review 9.1 *(For help, go to Section 8.1.)*

In Exercises 1 and 2, find the first four terms and the 30th term of the sequence

$$\{u_n\}_{n=1}^{\infty} = \{u_1, u_2, \ldots, u_n, \ldots\}.$$

1. $u_n = \dfrac{4}{n+2}$ **2.** $u_n = \dfrac{(-1)^n}{n}$

In Exercises 3 and 4, the sequences are geometric ($a_{n+1}/a_n = r$, a constant). Find

(a) the common ratio r. **(b)** the tenth term.

(c) a rule for the nth term.

3. $\{2, 6, 18, 54, \ldots\}$ **4.** $\{8, -4, 2, -1, \ldots\}$

In Exercises 5–10,

(a) graph the sequence $\{a_n\}$.

(b) determine $\lim_{n\to\infty} a_n$.

5. $a_n = \dfrac{1-n}{n^2}$ **6.** $a_n = \left(1 + \dfrac{1}{n}\right)^n$

7. $a_n = (-1)^n$ **8.** $a_n = \dfrac{1-2n}{1+2n}$

9. $a_n = 2 - \dfrac{1}{n}$ **10.** $a_n = \dfrac{\ln(n+1)}{n}$

Section 9.1 Exercises

1. Replace the $*$ with an expression that will generate the series

$$1 - \frac{1}{4} + \frac{1}{9} - \frac{1}{16} + \cdots.$$

(a) $\sum_{n=1}^{\infty} (-1)^{n-1}\left(\dfrac{1}{*}\right)$ **(b)** $\sum_{n=0}^{\infty} (-1)^n\left(\dfrac{1}{*}\right)$

(c) $\sum_{n=*}^{\infty} (-1)^n\left(\dfrac{-1}{(n-2)^2}\right)$

2. Write an expression for the nth term, a_n.

(a) $\sum_{n=0}^{\infty} a_n = 1 + \dfrac{1}{3} + \dfrac{1}{9} + \dfrac{1}{27} + \dfrac{1}{81} + \cdots$

(b) $\sum_{n=1}^{\infty} a_n = 1 - \dfrac{1}{2} + \dfrac{1}{3} - \dfrac{1}{4} + \dfrac{1}{5} - \cdots$

(c) $\sum_{n=0}^{\infty} a_n = 5 + 0.5 + 0.05 + 0.005 + 0.0005 + \cdots$

In Exercises 3–6, tell whether the series is the same as

$$\sum_{n=1}^{\infty}\left(-\frac{1}{2}\right)^{n-1}.$$

3. $\sum_{n=1}^{\infty} -\left(\dfrac{1}{2}\right)^{n-1}$ **4.** $\sum_{n=0}^{\infty}\left(-\dfrac{1}{2}\right)^n$

5. $\sum_{n=0}^{\infty} (-1)^n\left(\dfrac{1}{2}\right)^n$ **6.** $\sum_{n=1}^{\infty} \dfrac{(-1)^n}{2^{n-1}}$

In Exercises 7–10, compute the limit of the partial sums to determine whether the series converges or diverges.

7. $1 + 1.1 + 1.11 + 1.111 + 1.1111 + \cdots$

8. $2 - 1 + 1 - 1 + 1 - 1 + \cdots$

$\dfrac{1}{2} + \dfrac{1}{4} + \dfrac{1}{8} + \cdots + \dfrac{1}{2^k} + \cdots$

10. $3 + 0.5 + 0.05 + 0.005 + 0.0005 + \cdots$

In Exercises 11–20, tell whether the series converges or diverges. If it converges, give its sum.

11. $1 + \dfrac{2}{3} + \left(\dfrac{2}{3}\right)^2 + \left(\dfrac{2}{3}\right)^3 + \cdots + \left(\dfrac{2}{3}\right)^n + \cdots$

12. $1 - 2 + 3 - 4 + 5 - \cdots + (-1)^n(n+1) + \cdots$

13. $\sum_{n=0}^{\infty}\left(\dfrac{5}{4}\right)\left(\dfrac{2}{3}\right)^n$ **14.** $\sum_{n=0}^{\infty}\left(\dfrac{2}{3}\right)\left(\dfrac{5}{4}\right)^n$

15. $\sum_{n=0}^{\infty} \cos(n\pi)$

16. $3 - 0.3 + 0.03 - 0.003 + 0.0003 - \cdots + 3(-0.1)^n + \cdots$

17. $\sum_{n=0}^{\infty} \sin^n\left(\dfrac{\pi}{4} + n\pi\right)$

18. $\dfrac{1}{2} + \dfrac{2}{3} + \dfrac{3}{4} + \dfrac{4}{5} + \cdots + \dfrac{n}{n+1} + \cdots$

19. $\sum_{n=1}^{\infty}\left(\dfrac{e}{\pi}\right)^n$ **20.** $\sum_{n=0}^{\infty} \dfrac{5^n}{6^{n+1}}$

In Exercises 21–24, find the interval of convergence and the function of x represented by the geometric series.

21. $\sum_{n=0}^{\infty} 2^n x^n$ **22.** $\sum_{n=0}^{\infty} (-1)^n(x+1)^n$

23. $\sum_{n=0}^{\infty}\left(-\dfrac{1}{2}\right)^n(x-3)^n$ **24.** $\sum_{n=0}^{\infty} 3\left(\dfrac{x-1}{2}\right)^n$

In Exercises 25 and 26, find the values of x for which the geometric series converges and find the function of x it represents.

25. $\sum_{n=0}^{\infty} \sin^n x$ **26.** $\sum_{n=0}^{\infty} \tan^n x$

In Exercises 27–30, use the series and the function $f(x)$ that it represents from the indicated exercise to find a power series for $f'(x)$.

27. Exercise 21 **28.** Exercise 22

29. Exercise 23 **30.** Exercise 24

In Exercises 31–34, use the series and the function $f(x)$ that it represents from the indicated exercise to find a power series for $\int_0^x f(t)\,dt$.

31. Exercise 21 **32.** Exercise 22

33. Exercise 23 **34.** Exercise 24

35. Writing to Learn Each of the following series diverges in a slightly different way. Explain what is happening to the sequence of partial sums in each case.

(a) $\sum_{n=1}^{\infty} 2n$ (b) $\sum_{n=0}^{\infty} (-1)^n$ (c) $\sum_{n=1}^{\infty} (-1)^n(2n)$

36. Prove that $\sum_{n=0}^{\infty} \frac{e^{n\pi}}{\pi^{ne}}$ diverges.

37. Solve for x: $\sum_{n=0}^{\infty} x^n = 20.$

38. Writing to Learn Explain how it is possible, given any real number at all, to construct an infinite series of non-zero terms that converges to it.

39. Make up a geometric series $\sum ar^{n-1}$ that converges to the number 5 if

(a) $a = 2$ (b) $a = 13/2$

In Exercises 40 and 41, express the repeating decimal as a geometric series and find its sum.

40. $0.\overline{21}$ **41.** $0.\overline{234}$

In Exercises 42–47, express the number as the ratio of two integers.

42. $0.\overline{7} = 0.7777\ldots$

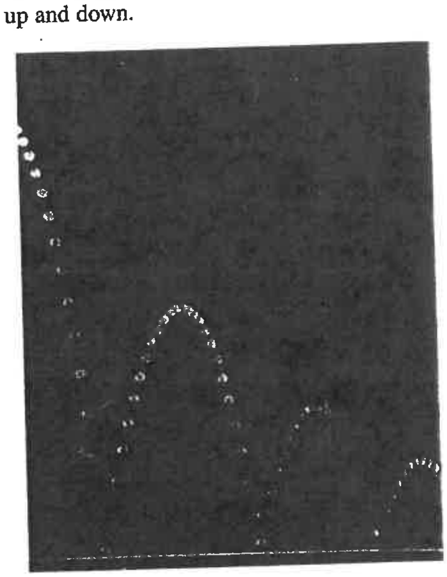
43. $0.\overline{d} = 0.dddd\ldots,$ where d is a digit

44. $0.0\overline{6} = 0.06666\ldots$ **45.** $1.\overline{414} = 1.414\ 414\ 414\ldots$

46. $1.24\overline{123} = 1.24\ 123\ 123\ 123\ldots$

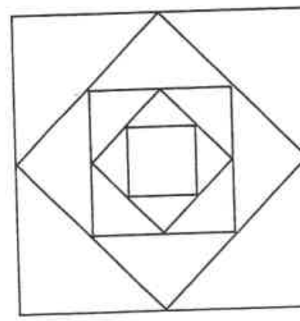
47. $3.\overline{142857} = 3.142857\ 142857\ldots$

48. *Bouncing Ball* A ball is dropped from a height of 4 m. Each time it strikes the pavement after falling from a height of h m, it rebounds to a height of $0.6h$ m. Find the total distance the ball travels up and down.

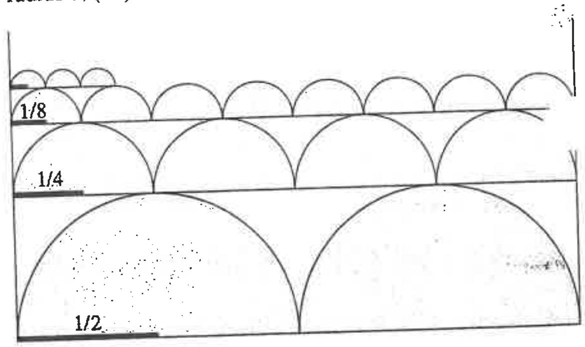


49. *(Continuation of Exercise 48)* Find the total number of seconds that the ball in Exercise 48 travels. (*Hint:* A freely falling ball travels $4.9t^2$ meters in t seconds, so it will fall meters in $\sqrt{h/4.9}$ seconds. Bouncing from ground to apex takes the same time as falling from apex to ground.)

50. *Summing Areas* The figure below shows the first five of an infinite sequence of squares. The outermost square has an area of 4 m^2. Each of the other squares is obtained by joining the midpoints of the sides of the preceding square. Find the sum of the areas of all the squares.



51. *Summing Areas* The accompanying figure shows the first three rows and part of the fourth row of a sequence of rows of semicircles. There are 2^n semicircles in the nth row, each of radius $1/(2^n)$. Find the sum of the areas of all the semicircles.



52. *Sum of a Finite Geometric Progression* Let a and r be real numbers with $r \neq 1$, and let

$$S = a + ar + ar^2 + ar^3 + \cdots + ar^{n-1}.$$

(a) Find $S - rS$.

(b) Use the result in part (a) to show that $S = \frac{a - ar^n}{1 - r}$.

53. *Sum of a Convergent Geometric Series* Exercise 52 gives a formula for the nth partial sum of an infinite geometric series. Use this formula to show that $\sum_{n=1}^{\infty} ar^{n-1}$ diverges whe $|r| \geq 1$ and converges to $a/(1 - r)$ when $|r| < 1$.

In Exercises 54–59, find a power series to represent the given functior and identify its interval of convergence. When writing the power serie include a formula for the nth term.

54. $\frac{1}{1 + 3x}$ **55.** $\frac{x}{1 - 2x}$

56. $\frac{3}{1 - x^3}$ **57.** $\frac{1}{1 + (x - 4)}$

58. $\frac{1}{4x} = \frac{1}{4}\left(\frac{1}{1 + (x - 1)}\right)$ **59.** $\frac{1}{2 - x}$ (*Hint:* Rewı