Quick Review 9.2 *(For help, go to Sections 3.3 and 3.6.)*

In Exercises 1–5, find a formula for the nth derivative of the function.

1. e^{2x}
2. $\frac{1}{x-1}$
3. 3^x
4. $\ln x$
5. x^n

In Exercises 6–10, find dy/dx. (Assume that letters other than x represent constants.)

6. $y = \frac{x^n}{n!}$
7. $y = \frac{2^n(x-a)^n}{n!}$
8. $y = (-1)^n \frac{x^{2n+1}}{(2n+1)!}$
9. $y = \frac{(x+a)^{2n}}{(2n)!}$
10. $y = \frac{(1-x)^n}{n!}$

Section 9.2 Exercises

In Exercises 1 and 2, construct the fourth order Taylor polynomial at $x = 0$ for the function.

1. $f(x) = \sqrt{1+x^2}$
2. $f(x) = e^{2x}$

In Exercises 3 and 4, construct the fifth order Taylor polynomial and the Taylor series for the function at $x = 0$.

3. $f(x) = \frac{1}{x+2}$
4. $f(x) = e^{1-x}$

In Exercises 5–12, use the table of Maclaurin series on the preceding page. Construct the first three nonzero terms and the general term of the Maclaurin series generated by the function and give the interval of convergence.

5. $\sin 2x$
6. $\ln(1-x)$
7. $\tan^{-1} x^2$
8. $7x e^x$
9. $\cos(x+2)$ (*Hint:* $\cos(x+2) = (\cos 2)(\cos x) - (\sin 2)(\sin x)$)
10. $x^2 \cos x$
11. $\frac{x}{1-x^3}$
12. e^{-2x}

In Exercises 13 and 14, find the Taylor series generated by the function at the given point.

13. $f(x) = \frac{1}{x+1}$, $x = 2$
14. $f(x) = e^{x/2}$, $x = 1$

In Exercises 15–17, find the Taylor polynomial of order 3 generated by f

(a) at $x = 0$; **(b)** at $x = 1$.

15. $f(x) = x^3 - 2x + 4$
16. $f(x) = 2x^3 + x^2 + 3x - 8$
17. $f(x) = x^4$

In Exercises 18–21, find the Taylor polynomials of orders 0, 1, 2, and 3 generated by f at $x = a$.

18. $f(x) = \frac{1}{x}$, $a = 2$
19. $f(x) = \sin x$, $a = \pi/4$
20. $f(x) = \cos x$, $a = \pi/4$
21. $f(x) = \sqrt{x}$, $a = 4$

22. Let f be a function that has derivatives of all orders for all real numbers. Assume $f(0) = 4$, $f'(0) = 5$, $f''(0) = -8$, and $f'''(0) = 6$.

(a) Write the third order Taylor polynomial for f at $x = 0$ and use it to approximate $f(0.2)$.

(b) Write the second order Taylor polynomial for f', the derivative of f, at $x = 0$ and use it to approximate $f'(0.2)$.

23. Let f be a function that has derivatives of all orders for all real numbers. Assume $f(1) = 4$, $f'(1) = -1$, $f''(1) = 3$, and $f'''(1) = 2$.

(a) Write the third order Taylor polynomial for f at $x = 1$ and use it to approximate $f(1.2)$.

(b) Write the second order Taylor polynomial for f', the derivative of f, at $x = 1$ and use it to approximate $f'(1.2)$.

24. The Maclaurin series for $f(x)$ is

$$f(x) = 1 + \frac{x}{2!} + \frac{x^2}{3!} + \frac{x^3}{4!} + \cdots + \frac{x^n}{(n+1)!} + \cdots.$$

(a) Find $f'(0)$ and $f^{(10)}(0)$.

(b) Let $g(x) = xf(x)$. Write the Maclaurin series for $g(x)$, showing the first three nonzero terms and the general term.

(c) Write $g(x)$ in terms of a familiar function without using series.

25. **(a)** Write the first three nonzero terms and the general term of the Taylor series generated by $e^{x/2}$ at $x = 0$.

(b) Write the first three nonzero terms and the general term of a power series to represent

$$g(x) = \frac{e^x - 1}{x}.$$

(c) For the function g in part (b), find $g'(1)$ and use it to show that

$$\sum_{n=1}^{\infty} \frac{n}{(n+1)!} = 1.$$

26. Let

$$f(t) = \frac{2}{1-t^2} \quad \text{and} \quad G(x) = \int_0^x f(t)\,dt.$$

(a) Find the first four terms and the general term for the Maclaurin series generated by f.

(b) Find the first four nonzero terms and the Maclaurin series for G.

27. **(a)** Find the first four nonzero terms in the Taylor series generated by $f(x) = \sqrt{1+x}$ at $x = 0$.

(b) Use the results found in part (a) to find the first four nonzero terms in the Taylor series for $g(x) = \sqrt{1+x^2}$ at $x = 0$.

(c) Find the first four nonzero terms in the Taylor series at $x = 0$ for the function h such that $h'(x) = \sqrt{1+x^2}$ and $h(0) = 5$.

28. Consider the power series

$$\sum_{n=0}^{\infty} a_n x^n, \text{ where } a_0 = 1 \text{ and } a_n = \left(\frac{3}{n}\right)a_{n-1} \text{ for } n \geq 1.$$

(This defines the coefficients *recursively*.)

(a) Find the first four terms and the general term of the series.

(b) What function f is represented by this power series?

(c) Find the exact value of $f'(1)$.

29. Use the technique of Exploration 3 to determine the number of terms of the Maclaurin series for cos x that are needed to approximate the value of cos 18 accurate to within 0.001 of the true value.

30. **Writing to Learn** Based on what you know about polynomial functions, explain why no Taylor polynomial of any order could actually equal sin x.

31. **Writing to Learn** Your friend has memorized the Maclaurin series for both sin x and cos x but is having a hard time remembering which is which. Assuming that your friend knows the trigonometric functions well, what are some tips you could give that would help match sin x and cos x with their correct series?

32. What is the coefficient of x^5 in the Maclaurin series generated by $\sin 3x$?

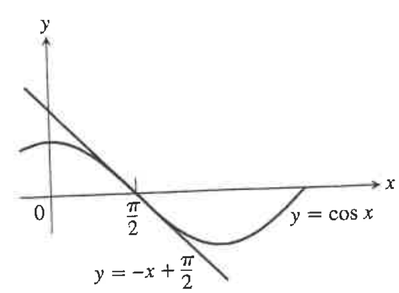
33. What is the coefficient of $(x-2)^3$ in the Taylor series generated by $\ln x$ at $x = 2$?

34. **Writing to Learn** Review the definition of the *linearization* of a differentiable function f at a in Chapter 4. What is the connection between the linearization of f and Taylor polynomials?

35. ***Linearizations at Inflection Points***

(a) As the figure below suggests, linearizations fit particularly well at inflection points. As another example, graph *Newton's serpentine* $f(x) = 4x/(x^2+1)$ together with its linearizations at $x = 0$ and $x = \sqrt{3}$.

(b) Show that if the graph of a twice-differentiable function $f(x)$ has an inflection point at $x = a$, then the linearization of f at $x = a$ is also the second order Taylor polynomial of f at $x = a$. This explains why tangent lines fit so well at inflection points.



The graph of $f(x) = \cos x$ and its linearization at $\pi/2$. (Exercise 35)

36. According to the table of Maclaurin series, the power series

$$x - \frac{x^3}{3} + \frac{x^5}{5} - \cdots + (-1)^n \frac{x^{2n+1}}{2n+1} + \cdots$$

converges at $x = \pm 1$. To what number does it converge when $x = 1$? To what number does it converge when $x = -1$?

Standardized Test Questions

You should solve the following problems without using a graphing calculator.

In Exercises 37 and 38, the Taylor series generated by $f(x)$ at $x = 0$ is

$$x - \frac{x^3}{3} + \frac{x^5}{5} - \cdots + (-1)^n \frac{x^{2n+1}}{2n+1} + \cdots.$$

37. **True or False** $f(0) = 0$. Justify your answer.

38. **True or False** $f'''(0) = -1/3$. Justify your answer.

39. **Multiple Choice** If $f(0) = 0$, $f'(0) = 1$, $f''(0) = 0$, and $f'''(0) = 2$, then which of the following is the third order Taylor polynomial generated by $f(x)$ at $x = 0$?

(A) $2x^3 + x$ (B) $\frac{1}{3}x^3 + \frac{1}{2}x$ (C) $\frac{2}{3}x^3 + x$

(D) $2x^3 - x$ (E) $\frac{1}{3}x^3 + x$

40. **Multiple Choice** Which of the following is the coefficient of x^4 in the Maclaurin series generated by cos $(3x)$?

(A) 27/8 (B) 9 (C) 1/24 (D) 0 (E) −27/8

In Exercises 41 and 42, let $f(x) = \sin x$.

41. **Multiple Choice** Which of the following is the fourth order Taylor polynomial generated by $f(x)$ at $x = \pi/2$?

(A) $(x - \pi/2) - \frac{(x-\pi/2)^2}{2!} + \frac{(x-\pi/2)^4}{4!}$

(B) $1 + \frac{(x-\pi/2)^2}{2!} + \frac{(x-\pi/2)^4}{4!}$

(C) $1 - \frac{(x-\pi/2)^2}{2!} + \frac{(x-\pi/2)^4}{4!}$

(D) $1 - (x-\pi/2)^2 + (x-\pi/2)^4$

(E) $1 + (x-\pi/2)^2 + (x-\pi/2)^4$

42. **Multiple Choice** Which of the following is the Taylor series generated by $f(x)$ at $x = \pi/2$?

(A) $\sum_{n=0}^{\infty} (-1)^n \frac{(x-\pi/2)^{2n}}{(2n)!}$

(B) $\sum_{n=0}^{\infty} (-1)^n \frac{(x-\pi/2)^{2n+1}}{(2n)!}$

(C) $\sum_{n=0}^{\infty} \frac{(x-\pi/2)^{2n}}{(2n)!}$

(D) $\sum_{n=0}^{\infty} (-1)^n (x-\pi/2)^{2n}$

(E) $\sum_{n=0}^{\infty} (x-\pi/2)^{2n}$