ection 9.4 Exercises

Exercises 1 and 2, find the values of x for which the equation is an entity. Support your answer graphically.

1. $\dfrac{-1}{x+4} = 1 + (x+5) + (x+5)^2 + (x+5)^3 + \cdots$

2. $\dfrac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots$

Exercises 3 and 4, use a comparison test to show that the series onverges for all x.

3. $\displaystyle\sum_{n=0}^{\infty} \frac{x^{3n}}{2n!+1}$ 4. $\displaystyle\sum_{n=0}^{\infty} \frac{x^{2n}}{n!+2}$

Exercises 5 and 6, show that the series converges absolutely.

5. $\displaystyle\sum_{n=0}^{\infty} \frac{(\cos x)^n}{n!+1}$ 6. $\displaystyle\sum_{n=0}^{\infty} \frac{2(\sin x)^n}{n!+3}$

In Exercises 7–22, find the *radius* of convergence of the power series.

7. $\displaystyle\sum_{n=0}^{\infty} x^n$ 8. $\displaystyle\sum_{n=0}^{\infty} (x+5)^n$

9 $(-1)^n(4x+1)^n$ 10. $\displaystyle\sum_{n=1}^{\infty} \frac{(3x-2)^n}{n}$

11. $\displaystyle\sum_{n=0}^{\infty} \frac{(x-2)^n}{10^n}$ 12. $\displaystyle\sum_{n=0}^{\infty} \frac{nx^n}{n+2}$

13. $\displaystyle\sum_{n=1}^{\infty} \frac{x^n}{n\sqrt{n}\,3^n}$ 14. $\displaystyle\sum_{n=0}^{\infty} \frac{x^{2n+1}}{n!}$

15. $\displaystyle\sum_{n=0}^{\infty} \frac{n(x+3)^n}{5^n}$ 16. $\displaystyle\sum_{n=0}^{\infty} \frac{nx^n}{4^n(n^2+1)}$

17. $\displaystyle\sum_{n=0}^{\infty} n!(x-4)^n$ 18. $\displaystyle\sum_{n=0}^{\infty} \frac{\sqrt{n}\,x^n}{3^n}$

19. $\displaystyle\sum_{n=0}^{\infty} (-2)^n(n+1)(x-1)^n$ 20. $\displaystyle\sum_{n=1}^{\infty} \frac{(4x-5)^{2n+1}}{n^{3/2}}$

21. $\displaystyle\sum_{n=1}^{\infty} \frac{(x+\pi)^n}{\sqrt{n}}$ 22. $\displaystyle\sum_{n=0}^{\infty} \frac{(x-\sqrt{2})^{2n+1}}{2^n}$

In Exercises 23–28, find the *interval* of convergence of the series and, within this interval, the sum of the series as a function of x.

23. $\displaystyle\sum_{n=0}^{\infty} \frac{(x-1)^{2n}}{4^n}$ 24. $\displaystyle\sum_{n=0}^{\infty} \frac{(x+1)^{2n}}{9^n}$

25. $\displaystyle\sum_{n=0}^{\infty} \left(\frac{\sqrt{x}}{2} - 1\right)^n$ 26. $\displaystyle\sum_{n=0}^{\infty} (\ln x)^n$

27 $\displaystyle\left(\frac{x^2-1}{3}\right)^n$ 28. $\displaystyle\sum_{n=0}^{\infty} \left(\frac{\sin x}{2}\right)^n$

In Exercises 29–44, determine the convergence or divergence of the series. Identify the test (or tests) you use. There may be more than one correct way to determine convergence or divergence of a given series.

29. $\displaystyle\sum_{n=1}^{\infty} \frac{n}{n+1}$ 30. $\displaystyle\sum_{n=1}^{\infty} \frac{2^n}{n+1}$

31. $\displaystyle\sum_{n=1}^{\infty} \frac{n^2-1}{2^n}$ 32. $\displaystyle\sum_{n=1}^{\infty} -\frac{1}{8^n}$

33. $\displaystyle\sum_{n=1}^{\infty} \frac{2^n}{3^n+1}$ 34. $\displaystyle\sum_{n=1}^{\infty} n\sin\left(\frac{1}{n}\right)$

35. $\displaystyle\sum_{n=0}^{\infty} n^2e^{-n}$ 36. $\displaystyle\sum_{n=0}^{\infty} \frac{n^{10}}{10^n}$

37. $\displaystyle\sum_{n=1}^{\infty} \frac{(n+3)!}{3!\,n!\,3^n}$ 38. $\displaystyle\sum_{n=1}^{\infty} \left(1+\frac{1}{n}\right)^n$

39. $\displaystyle\sum_{n=0}^{\infty} \frac{(-2)^n}{3^n}$ 40. $\displaystyle\sum_{n=1}^{\infty} n!e^{-n}$

41. $\displaystyle\sum_{n=1}^{\infty} \frac{3^n}{n^3 2^n}$ 42. $\displaystyle\sum_{n=1}^{\infty} \frac{n\ln n}{2^n}$

43. $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{(2n+1)!}$

44. $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{n^n}$ (*Hint:* If you do not recognize L, try recognizing the reciprocal of L.)

45. Give an example to show that the converse of the nth-Term Test is false. That is, $\sum a_n$ might diverge even though $\lim_{n\to\infty} a_n = 0$.

46. Find two convergent series $\sum a_n$ and $\sum b_n$ such that $\sum (a_n/b_n)$ diverges.

47. **Writing to Learn** We reviewed in Section 9.1 how to find the interval of convergence for the geometric series $\sum_{n=0}^{\infty} x^n$. Can we find the interval of convergence of a geometric series by using the Ratio Test? Explain.

In Exercises 48–54, find the sum of the telescoping series.

48. $\displaystyle\sum_{n=1}^{\infty} \frac{4}{(4n-3)(4n+1)}$ 49. $\displaystyle\sum_{n=1}^{\infty} \frac{6}{(2n-1)(2n+1)}$

50. $\displaystyle\sum_{n=1}^{\infty} \frac{40n}{(2n-1)^2(2n+1)^2}$ 51. $\displaystyle\sum_{n=1}^{\infty} \frac{2n+1}{n^2(n+1)^2}$

52. $\displaystyle\sum_{n=1}^{\infty} \left(\frac{1}{\sqrt{n}} - \frac{1}{\sqrt{n+1}}\right)$

53. $\displaystyle\sum_{n=1}^{\infty} \left(\frac{1}{\ln(n+2)} - \frac{1}{\ln(n+1)}\right)$

54. $\displaystyle\sum_{n=1}^{\infty} (\tan^{-1}(n) - \tan^{-1}(n+1))$