Review for Test on Sequences and Series

1. The second and fifth terms of a geometric sequence are -4 and ½, respectfully. Find
   a. the common ratio

   b. the first term

   c. and explicit rule for the nth term

   d. a recursive rule for the nth term

2. The first and third terms of an arithmetic sequence are -1 and 5, respectfully. What is the sixth term?

3. What is the limit of the sequence with nth term $a_n = nsin(\frac{3\pi}{n})$?

4. Which of the following is the limit of the sequence with nth term $a_n = (-1)^n \frac{3n+1}{n-2}$
   A. -3
   B. 0
   C. 2
   D. 3
   E. Diverges

5. If $f(x) = \sum_{n=0}^{\infty} 2^n x^n$, which of the following could be the interval of convergence?
   A. X=0 only
   B. (-1,1)
   C. (-2,2)
   D. (-.5,.5)
   E. All reals

6. If $f(x) = \sum_{n=0}^{\infty} a_n (x-c)^n$, then f''(c)=
   A. 0
   B. n(n-1) an
   C. $\sum_{n=1}^{\infty} na_n (x-c)^{n-1}$
   D. $\sum_{n=2}^{\infty} a_n$
   E. $\sum_{n=2}^{\infty} n(n-1)a_n (x-c)^{n-2}$

7. Which of the following series converge?

I. $\sum_{n=0}^{\infty}(1-\frac{2}{3})^n$ II. $\sum_{n=0}^{\infty}(1+\frac{4}{17})^n$ III. $\sum_{n=0}^{\infty}(1+\frac{1}{n})^n$

A. I only
B. II only
C. I and II only
D. I and III only
E. III only

8. What are the first three nonzero terms of the power series for $xe^{-x}$?

9. $\sum_{n=0}^{\infty}(\sin\frac{\pi}{6})^n =$

10. What is the radius of convergence for $\sum_{n=0}^{\infty}\frac{n+1}{2n+1}\frac{(x-3)^n}{2^n}$ ?

11. Let f be the function defined by $f(x)=\sum_{n=0}^{\infty}2(\frac{x+2}{3})^n$ for all values of x for which the series converges.

a. Find the radius of convergence for the series.

b. Find the function that the series represents.

12. Assume that f has derivatives of all orders for all real numbers x, f(0)=2, f'(0)=-1, f''(0)=6, and $f^3(0)$=12. Which of the following is the third order Maclaurin polynomial?
A. $2-x+3x^2+2x^3$
B. $2-x+6x^2+12x^3$
C. $2-.5x+3x^2+2x^3$
D. $-2+x-3x^2-2x^3$
E. $2-x+6x^2$

13. What is the Taylor series generated by f(x)=1/x at x=1?

14. What is the sum of the series $\sum_{n=0}^{\infty}\frac{\pi^n}{e^{2n}}$?

15. Which of the following series converge?

I. $\sum_{n=1}^{\infty}\frac{4}{\sqrt{n}}$ II. $\sum_{n=1}^{\infty}\frac{1}{(ln4)^n}$ III. $\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}$

A. I only
B. II only
C. III only
D. I and II only
E.II and III only

16. What is the sum of the telescoping series $\sum_{n=1}^{\infty} \frac{2}{(n+1)(n+2)}$ ?

17. Let $f(x) = \frac{1}{x+1}$

a. Find the first three terms and the general term for the Taylor series at x=1.

b. Find the interval of convergence for the series on part (a).

c. Find the third order polynomial for f at x=1 and use it to approximate f(0.5).

18. Let $f(x) = \sum_{n=0}^{\infty} \frac{nx^n}{2^n}$ .

a. Find the interval of convergence of the series.

b. Show that the first nine terms of the series are sufficient to approximate f(-1) with an error less than 0.01.

19. Let $f(x) = \frac{1}{x-2}$.

a. Write the first four terms and the general term of the Taylor series generated by f(x) at x=3.

b. Use the result from part (a) to find the first four terms and the general term of the series generated by $\ln|x-2|$ at x=3.

20. The approximation $e^x$=1+ x+ ($x^2$/2) is used when x is small. Use the Remainder Estimation Theorem to estimate the error when $|x| < 0.1$.

Review for Test on Sequences and Series

1. The second and fifth terms of a geometric sequence are -4 and ½, respectfully. Find
   a. the common ratio $-1/2$
   b. the first term $8$
   c. and explicit rule for the nth term $8\left(-\frac{1}{2}\right)^{n-1}$
   d. a recursive rule for the nth term $\left(-\frac{1}{2}\right)a_{n-1}$

2. The first and third terms of an arithmetic sequence are -1 and 5, respectfully. What is the sixth term? $\frac{5-(-1)}{2}=3 \quad -1+3(5)=14$

   $a+(n-1)d$

3. What is the limit of the sequence with nth term $a_n = n\sin(\frac{3\pi}{n})$?

   $\lim_{n\to\infty} = 3\pi$

   $n\sin\frac{3\pi}{n}$ $\quad \frac{\sin\frac{3\pi}{n}}{\frac{1}{n}}$ $\quad \frac{\cos\frac{3\pi}{n}\cdot\frac{-3\pi}{n^2}}{\frac{-1}{n^2}}$ $\quad 3\pi\cdot\cos\frac{3\pi}{n}$

4. Which of the following is the limit of the sequence with nth term $a_n = (-1)^n \frac{3n+1}{n-2}$
   A. -3
   B. 0
   C. 2
   D. 3
   E. Diverges

   E. $+1$ or $-1$

5. If $f(x) = \sum_{n=0}^{\infty} 2^n x^n$, which of the following could be the interval of convergence?
   A. X=0 only
   B. (-1,1)
   C. (-2,2)
   D. (-.5,.5)
   E. All reals

   $-1 < 2x < 1$

   $\sum_{n=0}^{\infty} a_n(x-c)^n$

   $a_0(x-c)^0 + a_1(x-c)^1 + a_2(x-c)^2 + \ldots$

   $a_0 + a_1(x-c) + a_2(x-c)^2$

   $f' = a_1 + 2a_2(x-c) + \ldots$

   $f'' = 2a_2 + \ldots$

6. If $f(x) = \sum_{n=0}^{\infty} a_n(x-c)^n$, then f''(c)=
   A. 0
   B. n(n-1) an
   C. $\sum_{n=0}^{\infty} na_n(x-c)^{n-1}$
   D. $\sum_{n=0}^{\infty} a_n$
   E. $\sum_{n=0}^{\infty} n(n-1)a_n(x-c)^{n-2}$ $n=2$

   $2$

   $a_n \cdot n(x-c)^{n-1}$

   $a_n \cdot n(n-1)(x-c)^{n-2}$

   $a_2 \cdot 2(2-1)(x-c)^0$

   $= 2a_2 + \ldots$

7. Which of the following series converge?

I. $\sum_{n=0}^{\infty}(1-\frac{2}{3})^n$ II. $\sum_{n=0}^{\infty}(1+\frac{4}{17})^n$ III. $\sum_{n=0}^{\infty}(1+\frac{1}{n})^n$

$\frac{1}{3}<1$ $>1$ D $1^{\infty}$ D $\ln$ $=e$ $\ln y = \lim_{n\to\infty} 1+\frac{1}{n}$ $e$ $e$

A. I only
B. II only
C. I and II only
D. I and III only
E. III only

$\lim(1+\frac{1}{n})^{\infty}$ $\ln y = n\ln(1+\frac{1}{n})$ $\frac{\ln(1+\frac{1}{n})}{\frac{1}{n}}$ $\frac{\frac{\frac{-1}{n^2}}{1+\frac{1}{n}}}{-\frac{1}{n^2}}$

8. What are the first three nonzero terms of the power series for $xe^{-x}$?

$e^x = \sum \frac{x^n}{n!}$

$e^{-x} = \sum \frac{(-x)^n}{n!} = \sum \frac{(-1)^n x^n}{n!}$

$xe^{-x} = \sum \frac{(-1)^n x^{n+1}}{n!}$

9. $\sum_{n=0}^{\infty}(\sin\frac{\pi}{6})^n =$ $(\frac{1}{2})^n$ $a=1$ $r=\frac{1}{2}$ $\frac{1}{1-\frac{1}{2}}=2$

10. What is the radius of convergence for $\sum_{n=0}^{\infty}\frac{n+1}{2n+1}\frac{(x-3)^n}{2^n}$?

$\frac{n+2}{2n+3}\frac{(x-3)^{n+1}}{2^{n+1}} \cdot \frac{2^n}{n+1}\frac{2n+1}{x-3^n}$ $\frac{x-3}{2}$ $\left|\frac{x-3}{2}\right|<1$ $|x-3|<2$ $\boxed{R=2}$ $x = x - \frac{x^2}{1!}+\frac{x^3}{2!}$

11. Let f be the function defined by $f(x)=\sum_{n=0}^{\infty}2(\frac{x+2}{3})^n$ for all values of x for which the series converges.

a. Find the radius of convergence for the series. $\left|\frac{x+2}{3}\right|<1$ $\boxed{R=3}$

b. Find the function that the series represents. $r=\frac{x+2}{3}$ $a=2$ $\frac{2}{1-\frac{x+2}{3}}$ $\frac{2}{\frac{3-x-2}{3}}$ $\frac{2}{\frac{1-x}{3}}$ $\boxed{\frac{6}{1-x}}$

12. Assume that f has derivatives of all orders for all real numbers x, f(0)=2, f'(0)=-1, f''(0)=6, and $f^3(0)$=12. Which of the following is the third order Maclaurin series?

A. $2-x+3x^2+2x^3$
B. $2-x+6x^2+12x^3$
C. $2-.5x+3x^2+2x^3$
D. $-2+x-3x^2-2x^3$
E. $2-x+6x^2$

$2-x+\frac{6x^2}{3!}+\frac{12x^3}{4!}$

13. What is the Taylor series generated by f(x)=1/x at x=1?

$\frac{1}{x}$ $-\frac{1}{x^2}$ $\frac{2}{x^3}$ $-\frac{6}{x^4}$

$f(1)=1$ $f'(1)=-1$ $f''(1)=2$ $f'''(1)=-6$ $1-x+\frac{2(x-1)^2}{2!}-\frac{6(x-1)^3}{3!}$ 0 1 2 3

$\sum_{n=0}^{\infty}(-1)^n(x-1)^n$

14. What is the sum of the series $\sum_{n=0}^{\infty}\frac{\pi^n}{e^{2n}}$?

$\frac{e^2}{e^2-\pi}$

15. Which of the following series converge?

I. $\sum_{n=1}^{\infty} \frac{4}{\sqrt{n}}$ $\frac{4}{n^{1/2}}$ $\frac{1}{2} < 1$ D by p-series

II. $\sum_{n=1}^{\infty} \frac{1}{(ln4)^n} = 0$ $r = \frac{1}{\ln 4} < 1$ C

III. $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$ C $\frac{|-1^n|}{n^2} \le \frac{1}{n^2}$ $\frac{1}{n^2}$ C

A. I only

B. II only

C. III only

D. I and II only

E. II and III only

$2 = A(n+2) + B(n+1)$

$2 = A$ $n = -1$

16. What is the sum of the telescoping series $\sum_{n=1}^{\infty} \frac{2}{(n+1)(n+2)}$?

$\frac{2}{n+1} - \frac{2}{n+2} =$

$1 - \frac{2}{3}$

$(1-\frac{2}{3}) + (\frac{2}{3} - \frac{2}{4}) = 1 - \frac{2}{4}$

$(1-\frac{2}{3}) + (\frac{2}{3} - \frac{2}{4}) + (\frac{2}{4} - \frac{2}{5}) = 1 - \frac{2}{5}$

$S_n = 1 - \frac{2}{n+2}$

$S_n = \lim_{n\to\infty} 1 - \frac{2}{n+2} = 1$

17. Let $f(x) = \frac{1}{x+1}$

$f(1) = \frac{1}{2}$

$f'(x) = -(x+1)^{-2}$

$f'(1) = -\frac{1}{4}$

$f''(x) = 2(x+1)^{-3}$

$\frac{f''(1)}{2!} = \frac{1}{8}$

$f'''(x) = -6(x+1)^{-4}$

$\frac{f'''(1)}{3!} = -\frac{1}{16}$

$\frac{1}{2}\left(\frac{-1(x-1)}{2}\right)^n$ r

a. Find the first three terms and the general term for the Taylor series at x=1.

$\frac{f^n}{n!} = \frac{(-1)^n}{2^{n+1}}$ $f(x) = \frac{1}{2} - \frac{x-1}{4} + \frac{(x-1)^2}{8} + \ldots (-1)^n \frac{(x-1)^n}{2^{n+1}} + \ldots$

b. Find the interval of convergence for the series on part (a).

$\frac{1}{2} \cdot \left(\frac{-(x-1)}{2}\right)^n$ $\left|\frac{-x+1}{2}\right| < 1$ $|x-1| < 2$ $\boxed{-1 < x < 3}$

$2^n \cdot 2$

$-2 < -x + 1 < 2$

$-3 < -x < 1$

$3 > x > -1$

c. Find the third order polynomial for f at x=1 and use it to approximate f(0.5).

$\frac{1}{2} - \frac{x-1}{4} + \frac{(x-1)^2}{8} - \frac{1(x-1)^3}{16}$ $P_3(.5) \approx 0.664$

18. Let $f(x) = \sum_{n=0}^{\infty} \frac{nx^n}{2^n}$.

a. Find the interval of convergence of the series.

$\frac{n+1\,x^{n+1}}{2^{n+1}} \cdot \frac{2^n}{n x^n} = \frac{x}{2}$ $|x| < 2$ $(-2, 2)$

b. Show that the first nine terms of the series are sufficient to approximate f(-1) with an error less than 0.01.

$\frac{n(x)^n}{2^n}$ $\frac{n}{2^n}$ $(-1)$

trunc error

$-\frac{5}{2^5} + \frac{6}{2^6} - \frac{7}{2^7} + \frac{8}{2^8} - \frac{9}{2^9} + \frac{10}{2^{10}}$

$0 - \frac{1}{2} + \frac{2}{4} - \frac{3}{8} + \frac{4}{16} + \ldots (-1)\frac{n}{2^n} + \ldots$ $\lim_{n\to\infty}$

$S = \frac{a}{1-r}$

Plug in $x = -1$ $n = 10$ $\left|\frac{10(-1)^n}{2^{10}}\right| < .01$

$\left|\frac{10(-\frac{1}{2})^{10}}{1-(-\frac{1}{2})}\right| = |.0065| < .01$

19) a. $f(3) = (x-2)^{-1}\Big|_{x=3} = 1$

$f'(3) = -(x-2)^{-2} \rightarrow -1$

$f''(3) = 2(x-2)^{-3} \rightarrow \dfrac{f''(3)}{2!} = 1$

$f'''(3) = -6(x-2)^{-4} \rightarrow \dfrac{f'''(3)}{3!} = -1$

$f^{n}(3) = (-1)^n\, n! \qquad \dfrac{f^{n}(3)}{n!} = (-1)^n$

$f(x) = 1 - (x-3) + (x-3)^2 - (x-3)^3 + (-1)^n (x-3)^n$

b) $\ln|x-2| = \displaystyle\int_3^x \frac{1}{t-2}\,dt$

$= t - \frac{1}{2}(t-3)^2 + \frac{1}{3}(t-3)^3 - \frac{1}{4}(t-3)^4 + \dfrac{(-1)^n (t-3)^{n+1}}{n+1}$

$$\boxed{\dfrac{(-1)^n (x-3)^{n+1}}{n+1}}$$

$e^x = 1 + x + \frac{x^2}{2!}$

$e^{-x} = 1 + (-x) + \frac{(-x)^2}{2!}$

$xe^{-x} = x + x(-x) + \frac{x(-x)^2}{2!}$

$x - x^2 + \frac{x^3}{2!}$

20. $1 + x + \frac{x^2}{2} \quad P_2(x) \quad f^3(x) = e^x$

$e^{.1} \quad r=1 \quad |R_2(x)| = \dfrac{e^{.1}|x|^3}{3!} \quad \dfrac{e^{.1}(.1)^3}{3!} = 1.842\times10^{-4}$