# ection 9.5 Exercises

Exercises 1and 2, use the Integral Test to determine convergence or vergence of the series.

1. $\sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{n}}$

2. $\sum_{n=1}^{\infty} n^{-3/2}$

3. Find the first six partial sums of $\sum_{n=1}^{\infty} \frac{1}{n}$.

4. If $S_k$ is the $k$-th partial sum of $\sum_{n=1}^{\infty} \frac{1}{n}$, find the first value of $k$ for which $S_k > 4$.

Exercises 5 and 6, use the Limit Comparison Test to determine onvergence or divergence of the series.

5. $\sum_{n=1}^{\infty} \frac{3n-1}{n^2+1}$

6. $\sum_{n=0}^{\infty} \frac{2^n}{3^n+1}$

n Exercises 7–22, determine whether the series converges or diverges. There may be more than one correct way to determine convergence or divergence of a given series.

7. $\sum_{n=1}^{\infty} \frac{5}{n+1}$

8. $\sum_{n=1}^{\infty} \frac{3}{\sqrt{n}}$

9. $\sum_{n=2}^{\infty} \frac{\ln n}{n}$

10. $\sum_{n=1}^{\infty} \frac{1}{2n-1}$

11. $\sum_{n=1}^{\infty} \frac{1}{(\ln 2)^n}$

12. $\sum_{n=1}^{\infty} \frac{1}{(\ln 3)^n}$

13. $\sum_{n=1}^{\infty} n \sin\left(\frac{1}{n}\right)$

14. $\sum_{n=0}^{\infty} \frac{e^n}{1+e^{2n}}$

15. $\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n^2+1}$

16. $\sum_{n=1}^{\infty} \frac{5n^3-3n}{n^2(n+2)(n^2+5)}$

17. $\sum_{n=1}^{\infty} \frac{3^{n-1}+1}{3^n}$

18. $\sum_{n=2}^{\infty} (-1)^{n+1} \frac{1}{\ln n}$

19. $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{10^n}{n^{10}}$

20. $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{\sqrt{n}+1}{n+1}$

21. $\sum_{n=2}^{\infty} (-1)^{n+1} \frac{\ln n}{\ln n^2}$

22. $\sum_{n=1}^{\infty} \left(\frac{1}{n} - \frac{1}{n^2}\right)$

In Exercises 23–26, determine whether the series converges absolutely, converges conditionally, or diverges. Give reasons for your answer. Find a bound for the truncation error after 99 terms.

23. $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1+n}{n^2}$

24. $\sum_{n=1}^{\infty} (-1)^{n+1} (0.1)^n$

25. $\sum_{n=2}^{\infty} (-1)^{n+1} \frac{1}{n \ln n}$

26. $\sum_{n=1}^{\infty} (-1)^n n^2 \left(\frac{2}{3}\right)^n$

In Exercises 27–32, determine whether the series converges absolutely, converges conditionally, or diverges. Give reasons for your answers.

27. $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n!}{2^n}$

28. $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{\sin n}{n^2}$

29. $\sum_{n=1}^{\infty} \frac{(-1)^n}{1+\sqrt{n}}$

30. $\sum_{n=1}^{\infty} \frac{\cos n\pi}{n\sqrt{n}}$

31. $\sum_{n=1}^{\infty} \frac{\cos n\pi}{n}$

32. $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}+\sqrt{n+1}}$

In Exercises 33 and 34, show how to rearrange the terms of the series from the specified exercise to form **(a)** a divergent series, and **(b)** a series that converges to 4.

33. Exercise 23

34. Exercise 25

In Exercises 35–50, find **(a)** the *interval* of convergence of the series. For what values of $x$ does the series converge **(b)** absolutely, **(c)** conditionally?

35. $\sum_{n=0}^{\infty} x^n$

36. $\sum_{n=0}^{\infty} (x+5)^n$

37. $\sum_{n=0}^{\infty} (-1)^n (4x+1)^n$

38. $\sum_{n=1}^{\infty} \frac{(3x-2)^n}{n}$

39. $\sum_{n=0}^{\infty} \frac{(x-2)^n}{10^n}$

40. $\sum_{n=0}^{\infty} \frac{nx^n}{n+2}$

41. $\sum_{n=1}^{\infty} \frac{x^n}{n\sqrt{n}\,3^n}$

42. $\sum_{n=0}^{\infty} \frac{x^{2n+1}}{n!}$

43. $\sum_{n=0}^{\infty} \frac{n(x+3)^n}{5^n}$

44. $\sum_{n=0}^{\infty} \frac{nx^n}{4^n(n^2+1)}$

45. $\sum_{n=0}^{\infty} \frac{\sqrt{n}x^n}{3^n}$

46. $\sum_{n=0}^{\infty} n!(x-4)^n$

47. $\sum_{n=0}^{\infty} (-2)^n (n+1)(x-1)^n$

48. $\sum_{n=1}^{\infty} \frac{(4x-5)^{2n+1}}{n^{3/2}}$

49. $\sum_{n=1}^{\infty} \frac{(x+\pi)^n}{\sqrt{n}}$

50. $\sum_{n=0}^{\infty} (\ln x)^n$

51. Not only do the figures in Example 2 show that the $n$th partial sum of the harmonic series is less than $1 + \ln n$; they also show that it is *greater* than $\ln(n+1)$. Suppose you had started summing the harmonic series with $S_1 = 1$ at the time the universe was formed, 13 billion years ago. If you had been able to add a term every *second* since then, about how large would your partial sum be today? (Assume a 365-day year.)

52. **Writing to Learn** Write out a proof of the Integral Test (Theorem 10) for $N = 1$, explaining what you see in Figure 9.15.