CALCULUS BC

WORKSHEET ON PARAMETRIC EQUATIONS AND GRAPHING

Work these on notebook paper. Make a table of values and sketch the curve, indicating the direction of your graph. Then eliminate the parameter. Do not use your calculator.

1. $x = 2t + 1$ and $y = t - 1$

2. $x = 2t$ and $y = t^2$, $-1 \le t \le 2$

3. $x = 2 - t^2$ and $y = t$

4. $x = \sqrt{t+2}$ and $y = 3 - t$

5. $x = t - 2$ and $y = 1 - \sqrt{t}$

6. $x = 2t$ and $y = |t - 1|$

7. $x = t$ and $y = \dfrac{1}{t^2}$

8. $x = 2\cos t - 1$ and $y = 3\sin t + 1$

9. $x = 2\sin t - 1$ and $y = \cos t + 2$

10. $x = \sec t$ and $y = \tan t$

CALCULUS BC
WORKSHEET ON PARAMETRICS AND CALCULUS

Work these on **notebook paper**. Do not use your calculator.

On problems 1 – 5, find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$.

1. $x = t^2,\ y = t^2 + 6t + 5$
2. $x = t^2 + 1,\ y = 2t^3 - t^2$
3. $x = \sqrt{t},\ y = 3t^2 + 2t$
4. $x = \ln t,\ y = t^2 + t$
5. $x = 3\sin t + 2,\ y = 4\cos t - 1$

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6. A curve C is defined by the parametric equations $x = t^2 + t - 1,\ y = t^3 - t^2$.

(a) Find $\frac{dy}{dx}$ in terms of t.

(b) Find an equation of the tangent line to C at the point where $t = 2$.

7. A curve C is defined by the parametric equations $x = 2\cos t,\ y = 3\sin t$.

(a) Find $\frac{dy}{dx}$ in terms of t.

(b) Find an equation of the tangent line to C at the point where $t = \frac{\pi}{4}$.

On problems 8 – 10, find:

(a) $\frac{dy}{dx}$ in terms of t.

(b) all points of horizontal and vertical tangency

8. $x = t + 5,\ y = t^2 - 4t$

9. $x = t^2 - t + 1,\ y = t^3 - 3t$

10. $x = 3 + 2\cos t,\ y = -1 + 4\sin t$

On problems 11 - 12, a curve C is defined by the parametric equations given. For each problem, write an integral expression that represents the length of the arc of the curve over the given interval.

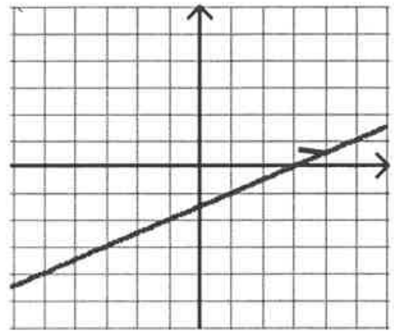
11. $x = t^2,\ y = t^3,\ 0 \le t \le 2$

12. $x = e^{2t} + 1,\ y = 3t - 1,\ -2 \le t \le 2$

Answers to Worksheet on Parametric Equations and Graphing

1. $x = 2t + 1$ and $y = t - 1$

t	-2	-1	0	1	2
x	-3	-1	1	3	5
y	-3	-2	-1	0	1



To eliminate the parameter, solve for $t = \frac{1}{2}x - \frac{1}{2}$.

Substitute into y's equation to get $y = \frac{1}{2}x - \frac{3}{2}$.

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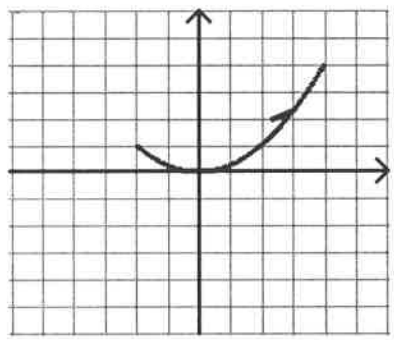
2. $x = 2t$ and $y = t^2$, $-1 \le t \le 2$

t	-1	0	1	2
x	-2	0	2	4
y	1	0	1	4



To eliminate the parameter, solve for $t = \frac{x}{2}$.

Substitute into y's equation to get

$y = \frac{x^2}{4}$, $-2 \le x \le 4$. Note: The restriction on x

is needed for the graph of $y = \frac{x^2}{4}$ to match the parametric graph.

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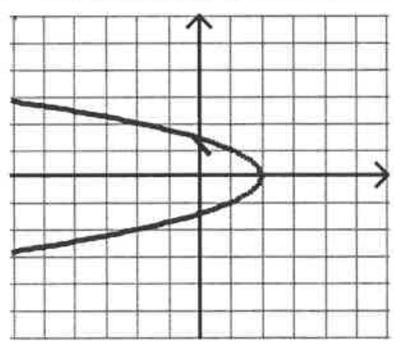
3. $x = 2 - t^2$ and $y = t$

t	-2	-1	0	1	2
x	-2	1	2	1	-2
y	-2	-1	0	1	2

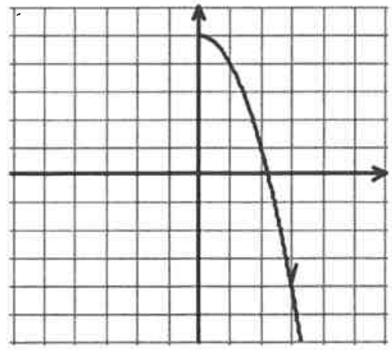


To eliminate the parameter, notice that $t = y$.
Substitute into x's equation to get
$x = 2 - y^2$.

4. $x=\sqrt{t+2}$ and $y=3-t$

t	-2	-1	2	7
x	0	1	2	3
y	5	4	1	-4

To eliminate the parameter, solve for $t=x^2-2$.
Substitute into y's equation to get
$y=5-x^2$, $x\geq 0$. Note: The restriction on x is
needed for the graph of $y=5-x^2$ to match the parametric graph.



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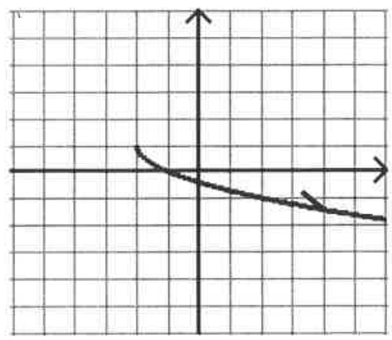
5. $x=t-2$ and $y=1-\sqrt{t}$

t	0	1	4	9
x	-2	-1	2	7
y	1	0	-1	-2

To eliminate the parameter, solve for $t=x+2$, $x\geq -2$
(since t ³ 0). Substitute into y's equation to get
$y=1-\sqrt{x+2}$.



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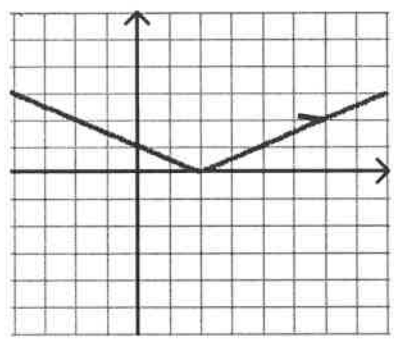
6. $x=2t$ and $y=|t-1|$

t	-2	-1	0	1	2	3
x	-1	-2	0	2	4	6
y	3	2	1	0	1	2

To eliminate the parameter, solve for $t=\frac{x}{2}$.

Substitute into y's equation to get
$y=\left|\frac{x}{2}-1\right|$ or $y=\frac{|x-2|}{2}$.



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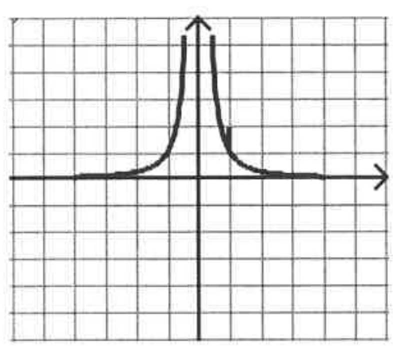
7. $x=t$ and $y=\frac{1}{t^2}$

t	-2	-1	$-1/2$	0	1/2	1	2
x	-2	-1	$-1/2$	0	1/2	1	2
y	1/4	1	4	und.	4	1	1/4

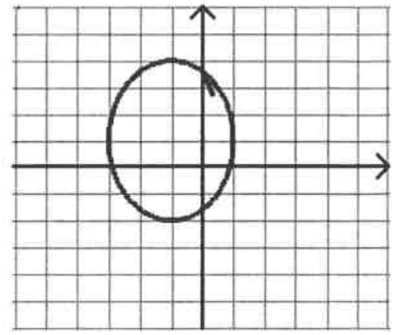
To eliminate the parameter, notice that $t=x$.

Substitute into y's equation to get $y=\frac{1}{x^2}$.



8. $x = 2\cos t - 1$ and $y = 3\sin t + 1$

t	0	$\pi/2$	π	$3\pi/2$	2π
x	1	-1	-3	-1	1
y	1	4	1	-2	1



To eliminate the parameter, solve for $\cos t$ in x's equation and $\sin t$ in y's equation. Substitute into the trigonometric identity

$\cos^2 t + \sin^2 t = 1$ to get $\dfrac{(x+1)^2}{4} + \dfrac{(y-1)^2}{9} = 1$.

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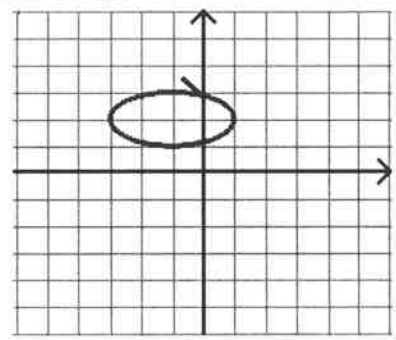
9. $x = 2\sin t - 1$ and $y = \cos t + 2$

t	0	$\pi/2$	π	$3\pi/2$	2π
x	-1	1	-1	-3	-1
y	3	2	1	2	3



To eliminate the parameter, solve for [ ] in y's equation and [ ] in x's equation. Substitute into the trigonometric identity

[ ] to get [ ].

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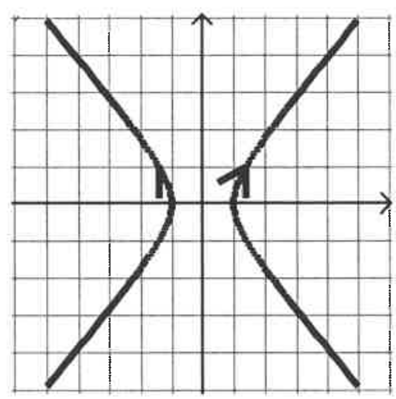
10. $x = \sec t$ and $y = \tan t$

t	0	$\pi/4$	$\pi/2$	$3\pi/4$	π	$5\pi/4$	$3\pi/2$	$7\pi/4$	2π
x	1	$\sqrt{2}$	und.	$-\sqrt{2}$	-1	$-\sqrt{2}$	und.	$\sqrt{2}$	1
y	0	1	und.	-1	0	1	und.	-1	0



To eliminate the parameter, substitute into the trigonometric identity $1 + \tan^2 t = \sec^2 t$ to get $1 + y^2 = x^2$ or $x^2 - y^2 = 1$.

Answers to Worksheet on Parametrics and Calculus

1. $\dfrac{dy}{dx}=\dfrac{2t+6}{2t}=1+\dfrac{3}{t};\quad \dfrac{d^2y}{dx^2}=\dfrac{-\dfrac{3}{t^2}}{2t}=-\dfrac{3}{2t^3}$

2. $\dfrac{dy}{dt}=3t-1;\quad \dfrac{d^2y}{dx^2}=\dfrac{3}{2t}$

3. $\dfrac{dy}{dx}=\dfrac{6t+2}{\frac{1}{2}t^{-\frac{1}{2}}}=12t^{\frac{3}{2}}+4t^{\frac{1}{2}};\quad \dfrac{d^2y}{dx^2}=\dfrac{18t^{\frac{1}{2}}+2t^{-\frac{1}{2}}}{\frac{1}{2}t^{-\frac{1}{2}}}=36t+4$

4. $\dfrac{dy}{dx}=\dfrac{2t+1}{\frac{1}{t}}=2t^2+t;\quad \dfrac{d^2y}{dx^2}=\dfrac{4t+1}{\frac{1}{t}}=4t^2+t$

5. $\dfrac{dy}{dx}=\dfrac{-4\sin t}{3\cos t}=-\dfrac{4}{3}\tan t;\quad \dfrac{d^2y}{dx^2}=\dfrac{-\frac{4}{3}\sec^2 t}{3\cos t}=-\dfrac{4}{9}\sec^3 t$

6. (a) $\dfrac{dy}{dx}=\dfrac{3t^2-2t}{2t+1}$ (b) $y-4=\dfrac{8}{5}(x-5)$

7. (a) $\dfrac{dy}{dx}=\dfrac{3\cos t}{-2\sin t}=-\dfrac{3}{2}\cot t$ (b) $y-\dfrac{3\sqrt{2}}{2}=-\dfrac{3}{2}\left(x-\sqrt{2}\right)$

8. (a) $\dfrac{dy}{dx}=\dfrac{2t-4}{1}$ (b) ~~Vert.~~ Horiz. tangent at $(7,-4)$. No point of ~~horiz.~~ Vert. tangency on this curve.

9. (a)

(b) Vert. tangent at the points $(1,-2)$ and $(3,2)$. Horiz. tangent at $\left(\dfrac{3}{4},-\dfrac{11}{8}\right)$.

10. (a) $\dfrac{dy}{dx}=\dfrac{4\cos t}{-2\sin t}=-2\cot t$

(b) Vert. tangent at $(3,3)$ and $(3,-5)$. Horiz. tangent at $(5,-1)$ and $(1,-1)$.

11. $s=\int_0^2\sqrt{4t^2+9t^4}\,dt$

12. $s=\int_{-2}^{2}\sqrt{4e^{4t}+9}\,dt$

CALCULUS BC
WORKSHEET 1 ON VECTORS

Work the following on **notebook paper**. Use your calculator on problems 10 and 13c only.

1. If $x = t^2 - 1$ and $y = e^{t^3}$, find $\frac{dy}{dx}$.

2. If a particle moves in the *xy*-plane so that at any time $t > 0$, its position vector is $\langle \ln(t^2 + 5t), 3t^2 \rangle$, find its velocity vector at time $t = 2$.

3. A particle moves in the *xy*-plane so that at any time t, its coordinates are given by $x = t^5 - 1$ and $y = 3t^4 - 2t^3$. Find its acceleration vector at $t = 1$.

4. If a particle moves in the *xy*-plane so that at time t its position vector is $\left\langle \sin\left(3t - \frac{\pi}{2}\right), 3t^2 \right\rangle$, find the velocity vector at time $t = \frac{\pi}{2}$.

5. A particle moves on the curve $y = \ln x$ so that its *x*-component has derivative $x'(t) = t + 1$ for $t \geq 0$. At time $t = 0$, the particle is at the point $(1, 0)$. Find the position of the particle at time $t = 1$.

6. A particle moves in the *xy*-plane in such a way that its velocity vector is $\langle 1 + t, t^3 \rangle$. If the position vector at $t = 0$ is $\langle 5, 0 \rangle$, find the position of the particle at $t = 2$.

7. A particle moves along the curve $xy = 10$. If $x = 2$ and $\frac{dy}{dt} = 3$, what is the value of $\frac{dx}{dt}$?

8. The position of a particle moving in the *xy*-plane is given by the parametric equations $x = t^3 - \frac{3}{2}t^2 - 18t + 5$ and $y = t^3 - 6t^2 + 9t + 4$. For what value(s) of t is the particle at rest?

9. A curve C is defined by the parametric equations $x = t^3$ and $y = t^2 - 5t + 2$. Write the equation of the line tangent to the graph of C at the point $(8, -4)$.

10. A particle moves in the *xy*-plane so that the position of the particle is given by $x(t) = 5t + 3\sin t$ and $y(t) = (8 - t)(1 - \cos t)$ Find the velocity vector at the time when the particle's horizontal position is $x = 25$.

11. The position of a particle at any time $t \geq 0$ is given by $x(t) = t^2 - 3$ and $y(t) = \frac{2}{3}t^3$.
    (a) Find the magnitude of the velocity vector at time $t = 5$.
    (b) Find the total distance traveled by the particle from $t = 0$ to $t = 5$. (c) Find $\frac{dy}{dx}$ as a function of x.

12. Point $P(x, y)$ moves in the *xy*-plane in such a way that $\frac{dx}{dt} = \frac{1}{t+1}$ and $\frac{dy}{dt} = 2t$ for $t \geq 0$.
    (a) Find the coordinates of P in terms of t given that $t = 1$, $x = \ln 2$, and $y = 0$.
    (b) Write an equation expressing y in terms of x.
    (c) Find the average rate of change of y with respect to x as t varies from 0 to 4.
    (d) Find the instantaneous rate of change of y with respect to x when $t = 1$.

13. Consider the curve C given by the parametric equations $x = 2 - 3\cos t$ and $y = 3 + 2\sin t$, for $-\frac{\pi}{2} \leq t \leq \frac{\pi}{2}$.
    (a) Find $\frac{dy}{dx}$ as a function of t. (b) Find the equation of the tangent line at the point where $t = \frac{\pi}{4}$.
    (c) The curve C intersects the *y*-axis twice. Approximate the length of the curve between the two *y*-intercepts.

Answers to Worksheet 1 on Vectors

1. $\frac{dy}{dx} = \frac{3t^2e^{t^3}}{2t} = \frac{3te^{t^3}}{2}$

2. $\left\langle \frac{9}{14}, 12 \right\rangle$

3. $\langle 20, 24 \rangle$

4. $\langle -3, 3\pi \rangle$

5. $\left(\frac{5}{2}, \ln\left(\frac{5}{2}\right) \right)$

6. $(9, 4)$

7. $-\frac{6}{5}$

8. $t = 3$

9. $y + 4 = -\frac{1}{12}(x - 8)$

10. $\langle 7.008, -2.228 \rangle$

11. (a) $\sqrt{2600}$ or $10\sqrt{26}$ (b) $\frac{2}{3}\left(26^{3/2} - 1\right)$ (c) $t = \sqrt{x+3}$

12. (a) $\left(\ln(t+1), t^2 - 1\right)$ (b) $y = \left(e^x - 1\right)^2 - 1$ or $y = e^{2x} - 2e^x$.

(c) $\frac{16}{\ln 5}$ (d) 4

13. (a) $\frac{2}{3}\cot t$ (b) $y - \left(3 + \sqrt{2}\right) = \frac{2}{3}\left(x - \left(2 - \frac{3\sqrt{2}}{2}\right)\right)$ (c) 3.756

AP CALCULUS BC Name:______________________

WORKSHEET ON VECTORS HOMEWORK

Work the following on **notebook paper**. Use your calculator on problems 7 – 11 only.

1. If $x = e^{2t}$ and $y = \sin(3t)$, find $\dfrac{dy}{dx}$ in terms of t.

2. Write an integral expression to represent the length of the path described by the parametric equations $x = \cos^3 t$ and $y = \sin^2 t$ for $0 \le t \le \dfrac{\pi}{2}$.

3. For what value(s) of t does the curve given by the parametric equations $x = t^3 - t^2 - 1$ and $y = t^4 + 2t^2 - 8t$ have a vertical tangent?

4. For any time $t \ge 0$, if the position of a particle in the xy-plane is given by $x = t^2 + 1$ and $y = \ln(2t+3)$, find the acceleration vector.

5. Find the equation of the tangent line to the curve given by the parametric equations $x(t) = 3t^2 - 4t + 2$ and $y(t) = t^3 - 4t$ at the point on the curve where $t = 1$.

6. If $x(t) = e^t + 1$ and $y = 2e^{2t}$ are the equations of the path of a particle moving in the xy-plane, write an equation for the path of the particle in terms of x and y.

7. A particle moves in the xy-plane so that its position at any time t is given by $x = \cos(5t)$ and $y = t^3$. What is the speed of the particle when $t = 2$?

8. The position of a particle at time $t \ge 0$ is given by the parametric equations $x(t) = \dfrac{(t-2)^3}{3} + 4$ and $y(t) = t^2 - 4t + 4$.
   (a) Find the magnitude of the velocity vector at $t = 1$.
   (b) Find the total distance traveled by the particle from $t = 0$ to $t = 1$.
   (c) When is the particle at rest? What is its position at that time?

9. An object moving along a curve in the xy-plane has position $(x(t), y(t))$ at time with $\dfrac{dx}{dt} = 1 + \tan(t^2)$ and $\dfrac{dy}{dt} = 3e^{\sqrt{t}}$. Find the acceleration vector and the speed of the object when $t = 5$.

10. A particle moves in the xy-plane so that the position of the particle is given by $x(t) = t + \cos t$ and $y(t) = 3t + 2\sin t$, $0 \le t \le \pi$. Find the velocity vector when the particle's vertical position is $y = 5$.

11. An object moving along a curve in the xy-plane has position $(x(t), y(t))$ at time t with $\dfrac{dx}{dt} = 2\sin(t^3)$ and $\dfrac{dy}{dt} = \cos(t^2)$ for $0 \le t \le 4$. At time $t = 1$, the object is at the position (3, 4).
    (a) Write an equation for the line tangent to the curve at (3, 4).
    (b) Find the speed of the object at time $t = 2$.
    (c) Find the total distance traveled by the object over the time interval $0 \le t \le 1$.
    (d) Find the position of the object at time $t = 2$.

12. A particle moving along a curve in the xy-plane has position $(x(t),\, y(t))$ at time t with

$\frac{dx}{dt} = \arcsin\left(\frac{t}{t+4}\right)$ and $\frac{dy}{dt} = \ln\left(t^2+3\right)$. At time $t = 1$, the particle is at the position (5, 6).

(a) Find the speed of the object at time $t = 2$.

(b) Find the total distance traveled by the object over the time interval $1 \le t \le 2$.

(c) Find $y(2)$.

(d) For $0 \le t \le 3$, there is a point on the curve where the line tangent to the curve has slope 8. At what time t, $0 \le t \le 3$, is the particle at this point? Find the acceleration vector at this point.

13. **2006 AP® CALCULUS BC FREE-RESPONSE QUESTIONS (Form B)**

2. An object moving along a curve in the xy-plane is at position $(x(t), y(t))$ at time t, where

$$\frac{dx}{dt} = \tan\left(e^{-t}\right) \text{ and } \frac{dy}{dt} = \sec\left(e^{-t}\right)$$

for $t \ge 0$. At time $t = 1$, the object is at position $(2, -3)$.

(a) Write an equation for the line tangent to the curve at position $(2, -3)$.

(b) Find the acceleration vector and the speed of the object at time $t = 1$.

(c) Find the total distance traveled by the object over the time interval $1 \le t \le 2$.

(d) Is there a time $t \ge 0$ at which the object is on the y-axis? Explain why or why not.

<u>Answers to Worksheet 2 on Vectors</u>

1. $\frac{3\cos(3t)}{2e^{2t}}$

2. $\int_0^{\pi/2} \sqrt{9\cos^4 t\sin^2 t + 4\sin^2 t\cos^2 t}\, dt$

3. $t = 0$ and $t = \frac{2}{3}$

4. $v(t) = \left\langle 2t, \frac{2}{2t+3} \right\rangle$, $a(t) = \left\langle 2, -\frac{4}{(2t+3)^2} \right\rangle$

5. $y+3 = -\frac{1}{2}(x-1)$

6. $y = 2x^2 - 4x + 2$.

7. 12.304

8. (a) $\sqrt{5}$ (b) 3.816 (c) At rest when $t = 2$. Position = (4, 0)

9. $a(5) = \langle 10.178,\ 6.277 \rangle$, speed = 28.083

10. $\langle 0.119,\ 3.944 \rangle$

11. (a) $y - 4 = 0.321(x-3)$ (b) 2.084 (c) 1.126 (d) (3.436, 3.557)

12. (a) 2.061 (b) 1.738 (c) 7.661 (d) $\langle 0.422,\ 0.179 \rangle$

13. AP Question – check online for ap solution

CALCULUS BC
WORKSHEET 3 ON VECTORS

Work the following on **notebook paper**. Use your calculator only on problems 3 – 7.

1. The position of a particle at any time $t \geq 0$ is given by $x(t) = t^2 - 2,\ y(t) = \frac{2}{3}t^3$.

(a) Find the magnitude of the velocity vector at $t = 2$.
(b) Set up an integral expression to find the total distance traveled by the particle from $t = 0$ to $t = 4$.
(c) Find $\frac{dy}{dx}$ as a function of x.
(d) At what time t is the particle on the y-axis? Find the acceleration vector at this time.

2. An object moving along a curve in the xy-plane has position $(x(t),\ y(t))$ at time t with the velocity vector $v(t) = \left(\frac{1}{t+1},\ 2t\right)$. At time $t = 1$, the object is at (ln 2, 4).

(a) Find the position vector.
(b) Write an equation for the line tangent to the curve when $t = 1$.
(c) Find the magnitude of the velocity vector when $t = 1$.
(d) At what time $t > 0$ does the line tangent to the particle at $(x(t),\ y(t))$ have a slope of 12?

3. A particle moving along a curve in the xy-plane has position $(x(t),\ y(t))$, with $x(t) = 2t + 3\sin t$ and $y(t) = t^2 + 2\cos t$, where $0 \leq t \leq 10$.

(a) Is the particle moving to the left or to the right when $t = 2.4$? Explain your answer.
(b) Find the velocity vector at the time when the particle's vertical position is $y = 7$.

4. A particle moving along a curve in the xy-plane has position $(x(t),\ y(t))$ at time t with $\frac{dx}{dt} = 1 + \sin\left(t^3\right)$. The derivative $\frac{dy}{dt}$ is not explicitly given. At time $t = 2$, the object is at position $(-5,\ 4)$.

(a) Find the x-coordinate of the position at time $t = 3$.
(b) For any $t \geq 0$, the line tangent to the curve at $(x(t),\ y(t))$ has a slope of $t + 3$. Find the acceleration vector of the object at time $t = 2$.

5. An object moving along a curve in the xy-plane has position $(x(t),\ y(t))$ at time t with $\frac{dx}{dt} = e^{\cos t}$ and $\frac{dy}{dt} = \sin\left(t^2\right)$ for $0 \leq t \leq 3$. At time $t = 3$, the object is at the point (1, 4).

(a) Find the equation of the tangent line to the curve at the point where $t = 3$.
(b) Find the speed of the object at $t = 3$.
(c) Find the total distance traveled by the object over the time interval $2 \leq t \leq 3$.
(d) Find the position of the object at time $t = 2$.

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6. A particle moving along a curve in the xy-plane has position $(x(t),\ y(t))$ at time t with $\frac{dx}{dt} = \sqrt{t^3+4}$ and $\frac{dy}{dt} = \cos^{-1}\left(e^{-t}\right)$. At time $t = 2$, the particle is at the point (5, 3).

(a) Find the acceleration vector for the particle at $t = 2$.
(b) Find the equation of the tangent line to the curve at the point where $t = 2$.
(c) Find the magnitude of the velocity vector at $t = 2$.
(d) Find the position of the particle at time $t = 1$.

7. An object moving along a curve in the xy-plane has position $(x(t),\ y(t))$ at time t with $\frac{dy}{dt} = 2 + \sin\left(e^t\right)$. The derivative $\frac{dx}{dt}$ is not explicitly given. At $t = 3$, the object is at the point (4, 5).

(a) Find the y-coordinate of the position at time $t = 1$.
(b) At time $t = 3$, the value of $\frac{dy}{dx}$ is -1.8. Find the value of $\frac{dx}{dt}$ when $t = 3$.
(c) Find the speed of the object at time $t = 3$.

Answers to Worksheet 3 on Vectors

1. (a) (b)

(c) $\frac{dy}{dx} = t = \sqrt{x+2}$ (d) $\langle 2,\ 4\sqrt{2} \rangle$

2. (a) $\left(\ln|t+1|,\ t^2+3\right)$ (b)

(c) $\frac{\sqrt{17}}{2}$ (d) $t = 2$

3. $\langle -0.968,\ 5.704 \rangle$

4. (a) -3.996 (b) $\langle -1.746,\ -6.741 \rangle$

5. (a) $y - 2 = 1.109(x - 3)$ (b) 0.555

(c) 0.878 (d) (0.529, 4.031)

6. (a) $\langle 1.732,\ 0.137 \rangle$ (b) $y - 3 = 0.414(x - 5)$

(c) 3.750 (d) (2.239, 1.664)

7. (a) 1.269 (b) (c) 3.368