1. $\displaystyle\sum_{k=0}^{\infty} \frac{3}{10^{k+1}} =$ (A) $\frac{3}{10}$ (B) $\frac{1}{3}$ (C) $\frac{3}{7}$ (D) $\frac{20}{33}$ (E) $\frac{6}{11}$

2. The first six terms in the sequence defined recursively by $a_1 = 1$, $a_2 = 1$, $a_n = a_{n-1} + a_{n-2}$ are
(A) {1,1,2,3,5,8} (B) {1,1,2,4,6,8} (C) {1,1,4,5,9,14} (D) {1,1,0,1,0,1}
(E) None of these

3. The formula for the n-th term of the series $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \ldots$ is

(A) $\frac{1}{2n+1}$ (B) $\frac{(-1)^n}{2n-1}$ (C) $\frac{(-1)^{n+1}}{n+2}$ (D) $\frac{(-1)^{n+1}}{4n-1}$ (E) $\frac{(-1)^{n+1}}{2n-1}$

4. Does the following converge or diverge? $\displaystyle\sum_{n=1}^{\infty} \frac{1}{(\sqrt{5}-1)^n}$

5. The integral test confirms that the series $\displaystyle\sum_{n=1}^{\infty} \frac{e^{1/n}}{n^2}$ converges. What is $\displaystyle\int_1^{\infty} \frac{e^{1/x}}{x^2}\,dx$?

(A) $e+1$ (B) $e-1$ (C) $\sqrt{e}-1$ (D) e^2 (E) $e^{1/4}-1$

6. **How many** of the series shown diverge? (i) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^n$ (ii) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}$ (iii) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$ (iv) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2}$

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

7. **How many** of the series shown converge? (i) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}$ (ii) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{e^n}$ (iii) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$ (iv) $\displaystyle\sum_{n=1}^{\infty} n$

(A) 1 (B) 2 (C) 3 (D) 4 (E) none of them converge

8. Find the sum of the series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{2^{(n-1)}}$.

9. Find the sum of the series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{3^{n-1}}$.

Answers: 1. B 2. A 3. E 4. converge 5. B 6. A 7. A 8. 2 9. 3/4

AP Calculus BC

CHAPTER 12A WORKSHEET

INFINITE SEQUENCES AND SERIES

Name______________________________

Seat # _______ Date ________________

Chapter 12A Review Sheet #2

Test each series for convergence or divergence. Identify the test used and show all your work.

1. $\displaystyle\sum_{n=1}^{\infty} \frac{n^2-1}{n^2+n}$

2. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2+n}$

3. $\displaystyle\sum_{k=1}^{\infty} \frac{(-3)^{k+1}}{2^{3k}}$

4. $\displaystyle\sum_{k=1}^{\infty} k^{-1.7}$

5. $\displaystyle\sum_{n=1}^{\infty} \frac{n}{e^n}$

6. $\displaystyle\sum_{n=2}^{\infty} \frac{2}{n(\ln n)^3}$

7. $\displaystyle\sum_{n=1}^{\infty} \frac{3^n n^2}{n!}$

8. $\displaystyle\sum_{j=1}^{\infty} \frac{3^j}{5^j+j}$

9. $\displaystyle\sum_{n=0}^{\infty} \frac{n!}{2\cdot 5\cdot 8\cdot \ldots \cdot (3n+2)}$

10. $\displaystyle\sum_{i=1}^{\infty} \frac{1}{\sqrt{i(i+1)}}$

11. $\displaystyle\sum_{n=1}^{\infty} (-1)^n 2^{1/n}$

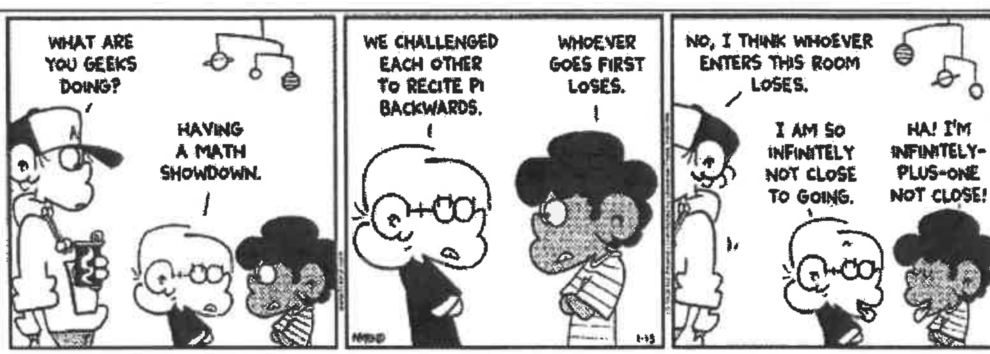
12. $\displaystyle\sum_{k=1}^{\infty} (-1)^k \frac{\ln k}{\sqrt{k}}$

13. $\displaystyle\sum_{n=1}^{\infty} \frac{(-2)^{2n}}{n^n}$

14. $\displaystyle\sum_{j=1}^{\infty} \frac{2^j}{(2j+1)!}$

15. $\displaystyle\sum_{n=1}^{\infty} \left(\sqrt[n]{2}-1\right)^n$

16. $\displaystyle\sum_{n=1}^{\infty} \sin n$



CALCULUS BC
WORKSHEET ON SERIES

Work the following on notebook paper. Use your calculator only on 10(b).

1. Which of the following is a term in the Taylor series about $x = 0$ for the function $f(x) = \cos(2x)$?

(A) $-\frac{1}{2}x^2$ (B) $-\frac{4}{3}x^3$ (C) $\frac{2}{3}x^4$ (D) $\frac{1}{60}x^5$ (E) $\frac{4}{45}x^6$

---

2. Find the values of x for which the series $\sum_{n=1}^{\infty}\frac{(x-2)^n}{n(-3)^n}$ converges.

(A) $x = 2$ (B) $-1 \le x < 5$ (C) $-1 < x \le 5$ (D) $-1 < x < 5$ (E) All real numbers

---

3. Let $f(x) = \sum_{n=1}^{\infty}(\cos x)^{3n}$. Evaluate $f\left(\frac{2\pi}{3}\right)$.

(A) $-\frac{1}{7}$ (B) $-\frac{1}{9}$ (C) $\frac{1}{7}$ (D) $\frac{8}{9}$ (E) The series diverges.

---

4. Find the sum of the geometric series $\frac{9}{8} - \frac{3}{4} + \frac{1}{2} - \frac{1}{3} + \ldots$

(A) $\frac{3}{5}$ (B) $\frac{5}{8}$ (C) $\frac{13}{24}$ (D) $\frac{27}{8}$ (E) $\frac{27}{40}$

---

5. The series $x + x^3 + \frac{x^5}{2!} + \frac{x^7}{3!} + \ldots + \frac{x^{2n+1}}{n!} + \ldots$ is the Maclaurin series for

(A) $x\ln\left(1+x^2\right)$ (B) $x\ln\left(1-x^2\right)$ (C) e^{x^2} (D) xe^{x^2} (E) $x^2e^{x^2}$

---

6. The coefficient of x^3 in the Taylor series for e^{2x} at $x = 0$ is

(A) $\frac{1}{6}$ (B) $\frac{1}{3}$ (C) $\frac{2}{3}$ (D) $\frac{4}{3}$ (E) $\frac{8}{3}$

---

7. The Taylor polynomial of order 3 at $x = 0$ for $f(x) = \sqrt{1+x}$ is

(A) $1 + \frac{x}{2} - \frac{x^2}{4} + \frac{3x^3}{8}$ (B) $1 + \frac{x}{2} - \frac{x^2}{8} + \frac{x^3}{16}$ (C) $1 - \frac{x}{2} + \frac{x^2}{8} - \frac{x^3}{16}$

(D) $1 + \frac{x}{2} - \frac{x^2}{8} + \frac{x^3}{8}$ (E) $1 - \frac{x}{2} + \frac{x^2}{4} - \frac{3x^3}{8}$

TURN->>>

8. The function f has a Taylor series about $x=2$ that converges to $f(x)$ for all x in the interval of convergence. The nth derivative of f at $x=2$ is given by $f^{(n)}(2)=\frac{(n+1)!}{3^n}$ for $n\geq 1$, and $f(2)=1$.

(a) Write the first four terms and the general term of the Taylor series for f about $x=2$.

(b) Find the radius of convergence for the Taylor series for f about $x=2$. Show the work that leads to your answer.

(c) Let g be a function satisfying $g(2)=3$ and $g'(x)=f(x)$ for all x. Write the first four terms and the general term of the Taylor series for g about $x=2$.

(d) Does the Taylor series for g as defined in part (c) converge at $x=-2$? Give a reason for your answer.

---

9. The function f is defined by the power series

$$f(x)=\sum_{n=0}^{\infty}\frac{(-1)^n x^{2n}}{(2n+1)!}=1-\frac{x^2}{3!}+\frac{x^4}{5!}-\frac{x^6}{7!}+\ldots+\frac{(-1)^n x^{2n}}{(2n+1)!}+\ldots$$

for all real numbers x.

(a) Find $f'(0)$ and $f''(0)$. Determine whether f has a local maximum, a local minimum, or neither at $x=0$. Give a reason for your answer.

(b) Show that the second-degree Taylor polynomial for f about $x=0$ approximates $f(1)$ with error less than $\frac{1}{100}$.

---

10. Let f be a function that has derivatives of all orders on the interval $(-1,1)$. Assume that $f(0)=6$, $f'(0)=8$, $f''(0)=30$, $f'''(0)=48$, and $\left|f^{(n)}(x)\right|\leq 75$ for all x in (0, 1).

(a) Write a third-degree Taylor polynomial for f about $x=0$.

(b) Use your answer to (a) to estimate the value of $f(0.2)$. What is the maximum possible error in making this estimate? Justify your answer.

201-NYB-05 - Calculus 2

REVIEW WORKSHEET FOR TEST #3

1. Find the general term of the following sequence, determine if it converges, and if so to what limit.

$$\frac{2}{1}, \frac{3}{3}, \frac{4}{5}, \frac{5}{7}, \ldots$$

2. Determine the convergence or divergence of the **sequences** given by the following general term a_n.

(a) $1+2\left(\frac{4}{5}\right)^n$ (b) $\frac{\ln(3/n^2)}{\ln(1/n)}$ (c) $\frac{2(-1)^n\sin(n^2)}{n+\ln(n)}$

3. Determine whether each **series** is convergent or divergent. If the series is convergent, find the sum.

(a) $\sum_{n=1}^{\infty}\frac{(-1)^n - e^{n+1}}{3^n}$

(b) $6+3+1.5+0.75+\ldots$

(c) $\sum_{n=1}^{\infty}\left(\frac{2}{3}+\frac{2}{5}\right)^n$

(d) $\sum_{n=1}^{\infty}\arctan(n+1)-\arctan(n)$

(e) $\sum_{n=1}^{\infty}\frac{4}{4n^2-1}$

4. Show that the series $\sum_{n=1}^{\infty}e^{-n}$ is convergent in **four different ways.**

5. Determine whether each **series** is convergent or divergent.

(a) $\sum_{n=1}^{\infty}\frac{(n!)^2}{(2n)!}$

(b) $\sum_{n=3}^{\infty}\frac{n^3-5n}{2n^5+n^4}$

(c) $\sum_{n=1}^{\infty}\frac{n!}{n3^n}$

(d) $\sum_{n=1}^{\infty}\frac{1}{(n+1)\sqrt{\ln(n+1)}}$

(e) $\sum_{n=1}^{\infty}\frac{n}{(n+1)3^{n+1}}$

(f) $\sum_{n=1}^{\infty}\left(\frac{3n}{2n+3}\right)^n$

(g) $\sum_{n=1}^{\infty}\frac{n!}{1\cdot 3\cdot 5\cdot 7\cdots(2n-1)\cdot(2n+1)}$

(h) $\sum_{n=1}^{\infty}\frac{5^n}{4^n-n}$

(i) $\sum_{n=0}^{\infty}\frac{2^n\arctan(n)^n}{e^n}$

(j) $\sum_{n=1}^{\infty}\left(1-\frac{2}{n^2}\right)$

(k) $\sum_{n=0}^{\infty}\frac{\sqrt{n+2}}{\sqrt{n^2+1}}$

(l) $\sum_{n=1}^{\infty}\tan(2^{-n})$

(m) $\sum_{n=2}^{\infty}\tan\left(\frac{\pi n^2+n}{1+4n^2}\right)$

(n) $\sum_{n=1}^{\infty}\frac{3}{n\sqrt{n}}$

6. For each alternating series below, determine if it is divergent, conditionally convergent or absolutely convergent.

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n (n+1)!}{2^{4n}}$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{\cos(\pi n)}{\sqrt[3]{n}+1}$

(b) $\displaystyle\sum_{n=2}^{\infty} \left(\frac{-1}{\ln(n)}\right)^n$

(d) $\displaystyle\sum_{n=2}^{\infty} (-1)^n \frac{n}{n^3+2n^2}$

7. Find the interval and radius of convergence for each power series.

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{x^n}{3n+1}$

(c) $\displaystyle\sum_{n=0}^{\infty} \frac{x^{2n}}{3^{n+1}}$

(b) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n (n+2)! x^n}{n!}$

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{(-2)^n (x+3)^n}{n}$

8. Find the MacLaurin series for the function $f(x) = \dfrac{1}{2+3x}$. Then, determine the interval and radius of convergence for the resulting series.

9. Find the first four nonzero terms in the Taylor series for the given function around the specified point.

(a) $f(x) = \sqrt{x}$ around $x = 4$ (b) $f(x) = \cos(\pi x)$ around $x = 1$

10. Suppose a_n is a positive, decreasing sequence such that the series $\sum a_n$ is convergent. Prove that the following series are also convergent.

(a) $\displaystyle\sum \frac{a_n}{1+a_n}$ (b) $\sum (a_n - \sin(a_n))$ (c) $\sum (-1)^n \tan(a_n)$